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INCOME COMPOSITION INEQUALITY*

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Abstract

The purpose of this paper is twofold. First, it introduces a novel inequality concept, named *income composition inequality*. Second, it constructs an indicator for its measurement. This paper argues that the study of income composition inequality across the income distribution allows for (i) novel political economy analysis of the evolution of economic systems and (ii) the technical assessment of the relationship between the functional and personal distribution of income. Following an empirical application on six European countries, this paper discusses possible avenues for future research on the matter, ranging from development issues to public finance.

JEL-Classification: C430, E250

Keywords: Income Inequality, Income Composition Inequality, Functional Income Distribution

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1 Introduction

The study of income distribution has been brought in from the cold. In his 1997 Presidential Address for the Royal Economic Society, Anthony Atkinson emphasised the need for the "re-incorporation of income distribution into the main body of economic analysis" (Atkinson, 1997, p. 297). Twenty years later, this Presidential Address can see its mark in the growing number of inequality studies produced throughout this period. Among this new surge of inequality research, Thomas Piketty's book *Capital in the XXI Century* features as one of the most important contributions (Piketty, 2014). By collecting a large historical database on the structure on income and wealth, Piketty studies the evolution of income and wealth distributions for three centuries and over more than twenty countries.[1] Among the several key facts about inequality dynamics that emerge from Piketty's work, I wish to emphasize one in particular. The rise in the top income shares in the US over the 1980-2010 period has been mainly driven by rising inequality in labor earnings. According to Piketty, this can be explained by two major factors: (i) rising inequality in access to skills and higher education and (ii) rising top managerial compensations (see also Piketty, 2015). The structure of inequality in the US nowadays is, therefore, considerably different from its structure before World War I, when high levels of inequality were mainly determined by an extreme concentration of capital incomes. This key fact teaches us an important lesson: similar levels of income inequality (like those in the US in 1930 and 2000)[2] can be characterized by completely different compositions of income sources, such as capital and labor incomes, across the income distribution. This fact draws attention to the analysis of another important, and until now missing dimension for distributional analysis: *inequality in income composition*. This paper aims at filling this gap by doing two things. First, it introduces in a formal setting the concept of income composition inequality across the income distribution. Second, it constructs a summary statistic, named *income-factor concentration index*, to measure the novel inequality concept proposed.

This paper argues that the study of income composition inequality is useful for two reasons. First, it allows for novel political economy analysis of the evolution of economic systems. In this respect, this article is closely related to the recent work by Milanovic

[1]The data collection work behind Piketty's book has been a cumulative effort of many scholars from the World Inequality Lab (Paris School of Economics).

[2]The top 10% income share in the US in 1930 and 2000 was approximately the 45% of national income in the economy.

(2017), in which a novel classification of economic systems is put forward. Second, it links the functional and personal distribution of income. For the latter reason, this work fits into the literature on the relationship between the functional and personal distribution of income.

The structure of this paper is the following. Section 2 reviews the literature on the relationship between the functional and personal distribution of income. Section 3 introduces in a formal setting the concept of income composition inequality. Section 4 constructs an indicator to measure income composition inequality. Section 5 derives the same indicator in a two-person economy and describes its usefulness and mathematical properties. Section 6 applies the proposed methodology to six European countries and discusses possible avenues for future research on the matter. Section 7 concludes.

2 Literature

The study of the relationship between the functional and personal distribution of income has seen a revival of interest over the past two decades (Atkinson, 2009, Piketty, 2014). Already in 1997, Atkinson argued that to understand the drivers of inequality, the economic theory of the distribution of income is in need of further development (Atkinson, 1997, p. 317). He argues that the current priority should be to bring the several existing contributions on this theory together into a single framework (p. 317). He also argues that, among the different aspects which affect the dynamics of the distribution of income, the relationship between the functional and personal distribution should feature prominently (p. 298).

Such relationship binds a macroeconomic phenomenon with a microeconomic one.[3] Brandolini (1992) claims that this link connects economic systems and people, and it is provided by what he calls "entitlement rules".[4] As Glyn (2011) points out, unfair entitlement rules may lead the employer's profit rate to grow faster than the employee's wage rate. Moreover, unfair entitlement rules are likely to trigger political tensions between different interest groups. Income inequality needs therefore to be analyzed with an eye for the multidimensional nature of the typologies of income. Unsurprisingly, the laws

[3] In a later article, Atkinson wrote that one reason for studying this link is that "there is at present and evident disjuncture between the macroeconomic measures of economic performance and the perceptions by citizens as to what is happening to their incomes?" (Atkinson, 2009, p. 5).

[4] According to Brandolini, the entitlement rules are "rules stating who has the right to receive a given type of income and which proportion of it" (Brandolini, 1992, p. 3).

which regulate the distribution were considered to be the principal problem in political economy by the classical author Ricardo (Ricardo, 1911).[5]

Several contributions have recently explored the empirical nature of the link between the functional and personal distribution. Piketty (2014) analyzes the long-run evolution of the functional distribution and of the top income shares at the international level. In his framework, Piketty considers top income shares as measures of income inequality.[6] His landmark book *Capital in the Twenty-First Century* (2014) is an attempt to combine the different data sources available, such as fiscal data, survey data and national accounts in a systematic way.[7] One of the most important findings from his research is that the capital share of income has increased in many developed countries over the last decades (see also Piketty, 2015). Furthermore, Piketty shows that the capital income share tends to move together with the capital-income ratio in the long run. Given that inequality in capital income is generally greater than inequality in labor income, the rising share of capital income in net product leads to a greater inter-personal inequality. This result highlights the positive relationship between the functional and personal distribution of income from a historical perspective.

Another empirical contribution on the matter is the article by Bengtsson and Waldenstrm (2018), who find evidence of a “strong, positive link [between the functional and personal distribution of income] that has grown stronger over the past century” by means of a novel historical cross-country database they personally assembled. However, they do not believe this relationship to remain stable over time, insofar as it can be contingent on production technology, the structure of personal income, and the institutional context. Francese and Mulas-Granados (2015), based on an analysis that covers up to 93 countries between 1970 and 2013, find instead that the distribution of income between labor and capital has not been a major factor in explaining income inequality. The two previous works provide evidence that, as Milanovic (2017) puts it, “the link is not as simple and unambiguous as it seems”.

[5] We report the famous statement by Ricardo, which says: “the produce of the earth - all that is derived from its surface by the united application of labour, machinery and capital, is divided among three classes of the community, namely, the proprietor of the land, the owner of the stock or capital necessary for its cultivation, and the labourers by whose industry it is cultivated . . . To determine the laws which regulate this distribution is the principal problem in Political Economy” (Ricardo, 1911 [1817], p. 1 in 1911 edition).

[6] The advantage of considering top income share as a measure of income inequality is that they can be easily compared both across countries and cross time.

[7] Piketty himself states that his book is primarily about the history of the distribution of income and wealth (Piketty, 2015).

On a technical level, few works have attempted to precisely measure the strength of this link. In his recent work, Milanovic (2017) argues that in a context of rising share of capital income the level of income inequality grows only under two conditions: (i) a high level of inequality in capital income and (ii) a high and positive association between capital-rich and overall income-rich people. These two conditions, operationalized by the Gini of capital income and the correlation coefficient between capital and total income respectively,[8] suggest an important theoretical connection between factor shares and income inequality. Particularly, the correlation coefficient between capital and total income, which is an elasticity of inter-personal income Gini to changes in capital income share, may act as an intuitive and simple measure of such link. However, this correlation coefficient does not formally determine the condition of transmission of changes in the functional distribution into income inequality, as it will be discussed later in the paper. Atkinson and Bourguignon (2000) and Atkinson (2009) approach the measurement of this link by decomposing the squared coefficient of variation of income, where there are two types of income: wage income and capital income.[9] In such way, they manage to show the condition under which an increase in the capital income share is transmitted into an increase in overall income inequality, as measured by the standard deviation of income. Another way of measuring the association between capital and labor have also recently been proposed by Atkinson and Lakner, 2017. The authors study the association between capital and labor by constructing a rank-based measure of association which is a discrete approximation of the copula density. All these methods, however, do not aim at precisely measuring the strength of this link, nor to create a single summary statistic for such purpose. Atkinson and Lakner, (2017), for instance, do not precisely discuss under which specific joint distributions of capital and labor the strength of the link is maximal and minimal. Furthermore, as it will be clear later in the paper, rank-based measures of associations are not suited to measure the strength of the link between the functional and personal distribution of income. On the other end, Atkinson and Bourguignon (2000) do not provide any summary statistic that can be used to measure the

[8]These two variables emerge from the Yitzhaki-Lerman decomposition of the Gini coefficient (Yitzhaki and Lerman, 1984).

[9]Specifically, the coefficient of variation of income V^2 can be written as a function of the capital share of income π, of the inequality of wage income V_w and capital income V_k, and of the correlation ρ between wage income and capital income: $V^2 = (1-\pi)^2 V_w^2 + \pi^2 V_k^2 + 2\pi(1-\pi)\rho V_w V_k$. Now, if we define λ as the relationship between wage income dispersion and capital income dispersion, then a rise in the capital share of income is transmitted into personal income inequality only when the following condition is satisfied: $\pi > \frac{1-\lambda\rho}{1+\lambda^2-2\lambda\rho}$.

strength of this relationship. As stated in the introduction, this paper argues that, in order to determine a formal link between these two distributions, we need to introduce a novel inequality concept, that we call *income composition inequality*. Then, by constructing an indicator of income composition inequality, it will be possible to measure the strength of such link.

3 Definition and Interpretation

We define income composition inequality in the following way:

Definition 3.1. *If we decompose total income into two factors, such as capital and labour income, then income composition inequality is the extent to which the income composition is distributed unevenly across the income distribution.*

From Definition 3.1 follows in a straightforward manner that inequality in income composition is *maximal* when individuals at the top and at the bottom of the income distribution separately earn the two different types of income, and *minimal* when each individual earns the same composition of the two factors.

While we use capital and labor as income sources in this paper, it is important to emphasize that the study of income composition inequality can be useful to analyze the joint distribution of *any pair* of income (or wealth) components, such as net income and taxes, saving and consumption, and financial and non-financial assets, among others.

A high level of income composition inequality is associated with a strong relationship between the functional and personal distribution of income. The underlying intuition is straightforward: if the rich earn all the capital income in the economy, then an increase in the capital income share rises the income of the rich. Analogous reasoning can be proposed to show that under a high level of income composition inequality the functional distribution of income can be seen as a measure of income inequality.

From a political economy perspective, the level of income composition inequality can provide us with insights on the "type of capitalism" of a given social system. Specifically, following the classification proposed by Milanovic (2017), under maximal inequality in income composition a society can be considered a case of *classical capitalism*, in which a group of rich individuals draws its income from capital, while a group of poor individuals draws its income from labor. On the contrary, under minimal inequality in income composition a society can be regarded as a case of *new capitalism*, or of a *multiple sources of income society*.[10] For instance, a reduction in income composition inequality suggests that the corresponding economic system is moving towards becoming a new form of capitalism, in which individuals have multiple sources of income at their disposal and where there is a weaker relationship between functional and personal distribution of income. In the next section, we introduce a statistical indicator to measure income composition inequality. This indicator is constructed by means of specific concentration curves (Kakwani, 1977a, 1977b), *the concentration curves for income source.*

4 Methodology

4.1 The Concentration Curve for Income Source

Suppose we have a fixed population of n individuals, each endowed with income Y_i with $i = 1, \ldots, n$. We can define each individual's income share as $y_i = \frac{Y_i}{Y}$, where $Y = \sum_{i=1}^{n} Y_i$ is the total income of the population. Total income is divided into two sources, capital (Π) and labor (W), so that $Y = \Pi + W$ and hence $y = 1 = \pi + w$, where $\pi = \frac{\Pi}{Y}$ and $w = \frac{W}{Y}$ are the capital and labor shares in income, respectively. Consider the following decomposition of individual i's income:

$$y_i = \alpha_i \pi + \beta_i w, \tag{1}$$

where $\alpha_i = \frac{\Pi_i}{\Pi}$ and $\beta_i = \frac{W_i}{W}$ are the relative shares of capital and labor of individual i, such that $\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \beta_i = 1$ and Π_i and W_i represent i's total amount of capital and labor. Assume that $y_i \leq y_{i+1}$ $\forall i = 1, \ldots, n-1$ and $y_0 = 0$, so that individuals are indexed by their income ranking. We can define $p = \frac{i}{n}$ as the proportion of the population with income less than or equal to y_p, so that $p \in \mathbb{Q} \coloneqq [0, 1]$. Let $\mathscr{L}(\mathbf{y}, p) = \sum_{j=1}^{i} y_j$, with $i = 1, \ldots, n$, be the Lorenz curve for income corresponding to the distribution $\mathbf{y}$.[11] We can define the concentration curve for capital, $\mathscr{L}(\boldsymbol{\pi}, p)$, corresponding to the distribution $\boldsymbol{\pi}$, as follows:

$$\mathscr{L}(\boldsymbol{\pi}, p) = \pi \sum_{j=1}^{i} \alpha_j \quad \forall i = 1, \ldots, n. \tag{2}$$

[10] To be perfectly in line with Milanovic's own framework, under minimal inequality in income composition we should refer to a *new capitalism 2*.

[11] We are defining the Lorenz curve here as in Shorrocks (1983).

Similarly, the concentration curve for labor, $\mathscr{L}(\mathbf{w}, p)$, corresponding to the distribution $\mathbf{w}$, is:

$$\mathscr{L}(\mathbf{w}, p) = w \sum_{j=1}^{i} \beta_j \quad \forall i = 1, \dots, n. \tag{3}$$

The two curves describe the cumulative distribution of capital and labor across the population with individuals being indexed by their income ranking. It is hence possible that an individual with a higher capital share be ranked below someone with a lower capital share, if the income of the latter is above that of the former (formally, we can find a pair (i, j) s.t. $\alpha_i > \alpha_j$ and $y_i < y_j$). Additionally, note that when $i \longrightarrow n$ (or $p \longrightarrow 1$) then $\mathscr{L}(\boldsymbol{\pi}, p) \longrightarrow \pi$ and $\mathscr{L}(\mathbf{w}, p) \longrightarrow w$, where $\pi, w \leq y$. The concentration curves for income source can also be regarded as *pseudo-Lorenz curves* (Fei et al., 1978) scaled down to the level of the related income share.
According to the previous decomposition of individual income, we can write as follows:

$$\mathscr{L}(\mathbf{y}, p) = \mathscr{L}(\boldsymbol{\pi}, p) + \mathscr{L}(\mathbf{w}, p) \quad \forall i = 1, \dots, n. \tag{4}$$

The Lorenz curve for income $\mathscr{L}(\mathbf{y}, p)$, for every p, can therefore be decomposed into the sum of the two previously-defined concentration curves. Now, we can write the Gini coefficient, $\mathscr{G}$, as follows:

$$\mathscr{G} = 1 - \frac{1}{n}\left(\sum_{i=1}^{n}\left[\mathscr{L}\left(\boldsymbol{\pi}, \frac{i}{n}\right) + \mathscr{L}\left(\boldsymbol{\pi}, \frac{i-1}{n}\right) + \mathscr{L}\left(w, \frac{i}{n}\right) + \mathscr{L}\left(w, \frac{i-1}{n}\right)\right]\right). \tag{5}$$

Figure 1 plots an example of $\mathscr{L}(\mathbf{y}, p)$ (the blue curve) and $\mathscr{L}(\boldsymbol{\pi}, p)$ (the red curve) for a population of size $n = 10$. Total income is equally split between capital and labor, hence $\pi = w = \frac{1}{2}$.

The concentration curves allow us to understand whether a given income source is concentrated primarily at the bottom or at the top of the income distribution. Given the interdependence of the two concentration curves (i.e., when one source is concentrated at the top the other is concentrated at the bottom), a single curve is sufficient to analyze the joint distribution of capital and labor. However, to precisely assess the extent to which capital and labor are polarized across the income distribution, two benchmark conditions must be defined: the *zero-* and *maximum-concentration conditions*. On the basis of these two conditions, the corresponding *zero-* and *maximum-concentration curves* are hence introduced.

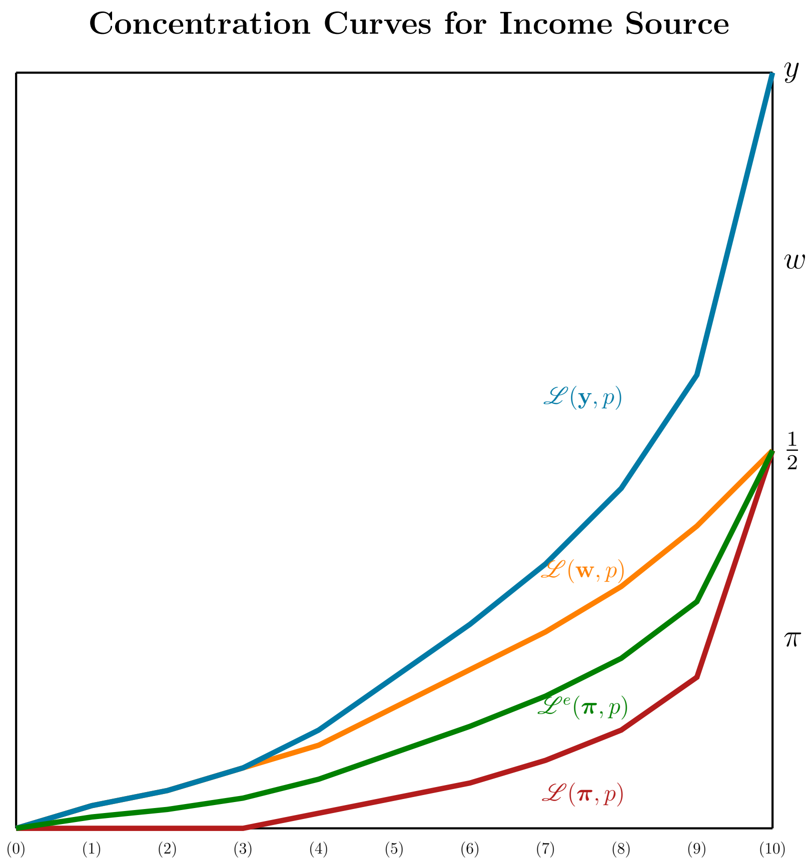


Figure 1: A graphical representation of the concentration curve for capital $\mathscr{L}(\boldsymbol{\pi},p)$, the concentration curve for labor $\mathscr{L}(\mathbf{w},p)$, the Lorenz curve for income $\mathscr{L}(\mathbf{y},p)$ and the zero-concentration curve $\mathscr{L}^e(\boldsymbol{\pi},p)$ with 10 individuals (or groups) and equal sources of income in the economy ($\pi = w = \frac{1}{2}$). As it can be noticed, for each population decile p the Lorenz curve for income $\mathscr{L}(\mathbf{y},p)$ equals the sum of the concentration curve for capital $\mathscr{L}(\boldsymbol{\pi},p)$ and the concentration curve for labor $\mathscr{L}(\mathbf{w},p)$. In addition, given that $\pi = w$, the two zero-concentration curves coincide: $\mathscr{L}^e(\boldsymbol{\pi},p) = \mathscr{L}^e(\mathbf{w},p)$ $\forall p$.

4.2 The Zero-Concentration Curve

In this Section, we introduce in a formal setting the concept of the zero concentration of two income sources. As anticipated in the introduction, we define the benchmark of zero concentration in the following way.

Definition 4.1. *We say that two income sources are zero-concentrated across a population when each individual has the same population shares of capital and labor. Formally, we have zero concentration of income sources when* $\frac{W_i}{\Pi_i} = \frac{w}{\pi}$ $\forall i$*, or, equivalently, when*

$\alpha_i = \beta_i \; \forall i$.[12]

Note that the previous definition is not related to the concept of income inequality: The population can exhibit zero concentration of income sources even with positive income inequality. Furthermore, note that only two elements are needed to determine the zero-concentration condition, notably the functional and personal distribution of income. Two populations characterized by different Lorenz curves, or by different shares of capital income, have two different conditions of zero-concentration. At this stage of the analysis we can define the zero-concentration curve, $\mathscr{L}^e(\mathbf{z}, p)$, corresponding to the distribution $\mathbf{z}$, which is the concentration curve for the income source z when the income sources are not concentrated as:

$$\mathscr{L}^e(\mathbf{z}, p) = \mathbf{z} \sum_{j=1}^{i} y_j \quad \forall i = 1, \ldots, n, \tag{6}$$

with $\mathbf{z} = \pi, w$. The choice of $\mathbf{z}$ depends on the particular source we analyze. If we were interested in the distribution of capital in the population, we would compare the *actual* concentration curve for capital with the concentration curve for capital in the case of zero concentration, $\mathscr{L}^e(\boldsymbol{\pi}, p)$. It should be noted that the zero-concentration curve is a *scaled* version of the Lorenz curve for income, indeed we can write $\mathscr{L}^e(\mathbf{z}, p) = \mathbf{z}\mathscr{L}(\mathbf{y}, p) \; \forall p$. Let us now consider the following relationship:

$$\mathscr{L}(\mathbf{z}, p) = \mathscr{L}^e(\mathbf{z}, p) + \mathscr{R}(\mathbf{z}, p) \quad \forall i = 1, \ldots, n, \tag{7}$$

where $\mathscr{R}(\mathbf{z}, p)$ is the residual-concentration curve corresponding to the distribution $\mathbf{z}$. When $\mathscr{L}(\mathbf{z}, p)$ is above $\mathscr{L}^e(\mathbf{z}, p)$ over all of the domain (i.e., $\mathscr{L}(\mathbf{z}, p) > \mathscr{L}^e(\mathbf{z}, p) \; \forall p$) then $\sum_{i=1}^{n} \mathscr{R}(\mathbf{z}, p) > 0$ and source $\mathbf{z}$ is concentrated primarily at the bottom of the distribution; on the contrary, when $\mathscr{L}(\mathbf{z}, p)$ is below $\mathscr{L}^e(\mathbf{z}, p)$ over all of the domain then $\sum_{i=1}^{n} \mathscr{R}(\mathbf{z}, p) < 0$ and the opposite situation holds. In the case of zero concentration of income sources, equation 5 becomes:

$$\mathscr{G} = 1 - \frac{1}{n} \left(\sum_{i=1}^{n} \left(\sum_{j=1}^{i} \beta_j + \sum_{j=1}^{i-1} \beta_j \right) \right), \tag{8}$$

which is also equivalent to:

$$\mathscr{G} = 1 - \frac{1}{n} \left(\sum_{i=1}^{n} \left(\sum_{j=1}^{i} \alpha_j + \sum_{j=1}^{i-1} \alpha_j \right) \right). \tag{9}$$

[12] As $\frac{W_i}{\Pi_i} = \frac{w}{\pi} \iff \frac{W_i}{w} = \frac{\Pi_i}{\pi} \iff Y \times \frac{W_i}{W} = Y \times \frac{\Pi_i}{\Pi} \iff \alpha_i = \beta_i$.

The Gini coefficient in this particular case can thus be written as a function of individuals' relative shares of any one income source. Note that none of the two expressions above are functions of π or w, meaning that an increase in either the capital share or the labor share of income does not affect personal income inequality when income sources are not concentrated. Similarly, we can say that the "elasticity of inter-personal income Gini to changes in capital income share" is zero.[13] This distribution of income sources represents the long-run distribution of factors across individuals in a neoclassical framework in which heterogeneity of both non-accumulated and accumulated factors are considered (Bertola et al. (2005)). It also represents the underlying distribution of factors in the *new capitalism 2* society defined by Milanovic (2017).

We conclude this Section with the following definition.

Definition 4.2. *We say that under zero-concentration of income sources, inequality in income composition is minimal.*

4.3 The Maximum-Concentration Curve

Let us focus our attention on the benchmark of maximum-concentration of two income sources, which we can define as follows.

Definition 4.3. *We say that two income sources are maximum concentrated when the bottom p% of the income distribution has an income consisting only of the source z and the top $(1-p)$% of the income distribution has an income consisting only of the source z_-, where p s.t. $y_p = \mathscr{L}(\boldsymbol{y}, p) = z$, $1-p$ s.t. $y_{1-p} = 1 - \mathscr{L}(\boldsymbol{y}, p) = z_-$, $z_- = 1 - z$ and $z = \pi, w$.*

As for the condition of zero-concentration, also the condition of maximum-concentration is already present in the literature. In his recent article, Milanovic defines the classical capitalism as a society in which "ownerships of capital and labor are totally separated, in the sense that workers draw their entire income from labor and have no income from the ownership of assets, while the situation for the capitalists is the reverse. Moreover, we shall assume that all workers are poorer than all capitalists. This gives us [...] two social groups, non-overlapping by income level" (Milanovic, 2017). We can therefore say

[13] See Milanovic (2017) for further details.

that under the condition of maximum-concentration and specifically when the capital is owned by the top of the income distribution and the labor by the bottom, a society is a classical capitalism á la Milanovic.[14]

From a technical perspective, we can define the maximum-concentration curve, $\mathscr{L}^{max}(\mathbf{z}, p)$, corresponding to the distribution $\mathbf{z}$, as follows:

$$\mathscr{L}^{max}(\mathbf{z}, p) = \begin{cases} \mathscr{L}^{M}(\mathbf{z}, p) = \begin{cases} \mathscr{L}(\mathbf{y}, p) & \text{for } p \leq p^{'} \\ z & \text{for } p > p^{'} \end{cases} \\ \mathscr{L}^{m}(\mathbf{z}, p) = \begin{cases} 0 & \text{for } p \leq p^{''} \\ \mathscr{L}(\mathbf{y}, p) - z_{-} & \text{for } p > p^{''} \end{cases}, \end{cases} \tag{10}$$

with $p^{'}$ s.t. $\mathscr{L}(\mathbf{y}, p^{'}) = z$, $p^{''}$ s.t. $\mathscr{L}(\mathbf{y}, p\grave{\imath}'') = 1 - z$ and $\mathbf{z} = \pi, w$. In addition, we have:

(i) $\mathscr{L}^{max}(\mathbf{z}, p) = \mathscr{L}^{M}(\mathbf{z}, p)$ if $\mathscr{L}(\mathbf{z}, p) \geq \mathscr{L}^{e}(\mathbf{z}, p)$ $\forall p$ and $\exists\ p^{*}$ s.t. $\mathscr{L}(\mathbf{z}, p^{*}) > \mathscr{L}^{e}(\mathbf{z}, p^{*})$,

(ii) $\mathscr{L}^{max}(\mathbf{z}, p) = \mathscr{L}^{m}(\mathbf{z}, p)$ if $\mathscr{L}^{e}(\mathbf{z}, p) \leq \mathscr{L}(\mathbf{z}, p)$ $\forall p$ and $\exists\ p^{**}$ s.t. $\mathscr{L}^{e}(\mathbf{z}, p^{**}) < \mathscr{L}(\mathbf{z}, p^{**})$.

To put it simply, $\mathscr{L}^{max}(\mathbf{z}, p) = \mathscr{L}^{M}(\mathbf{z}, p)$ when the actual concentration curve lies above the zero-concentration curve and that $\mathscr{L}^{max}(\mathbf{z}, p) = \mathscr{L}^{m}(\mathbf{z}, p)$ when the actual concentration curve lies below the zero-concentration curve.

However, the two conditions above-mentioned ((i) and (ii)) are rather strong, since they need the two curves not to intersect along the distribution of income. By contrast, a weaker condition is the one which takes into consideration the area covered by each curve, as follows:[15]

(i) $\mathscr{L}^{max}(\mathbf{z}, p) = \mathscr{L}^{M}(\mathbf{z}, p)$ if $\sum_{i=1}^{n} \sum_{j=1}^{i} \eta_j^k > \sum_{i=1}^{n} \sum_{j=1}^{i} y_j$,

(ii) $\mathscr{L}^{max}(\mathbf{z}, p) = \mathscr{L}^{m}(\mathbf{z}, p)$ if $\sum_{i=1}^{n} \sum_{j=1}^{i} \eta_j^k < \sum_{i=1}^{n} \sum_{j=1}^{i} y_j$,

where $\eta_j^k = \alpha_j$ if $z = \pi$ and $\eta_j^k = \beta_j$ when $z = w$.

As is the case for the previous Section, we conclude this Section with the following definition.

[14]This type of society can also be found in the works by Kaldor (1955), Pasinetti (1962) or more recently by Stiglitz (2015), in which a class of capitalists is counterposed to a class of workers. However, these authors do not necessarily assume that the former class is poorer than the latter in terms of total income.

[15]Similarly, the first and second group of conditions can be regarded as first and second-order stochastic dominance conditions.

Definition 4.4. *We say that under maximum-concentration of income sources, income composition inequality is maximized.*

4.4 Measuring Income Composition Inequality

In the previous Sections, we defined the two benchmarks of zero and maximum inequality in income composition, together with their corresponding concentration curves. When the actual concentration curve is relatively close the the zero-concentration curve, then income composition inequality is low. On the contrary, when the actual concentration curve is sufficiently close to the maximum-concentration curve, then income composition inequality is high.

To precisely measure income composition inequality, we introduce an indicator that serves this purpose, that we call *income-factor concentration index*. We label this indicator as $\mathscr{I}$, which is constructed in the following way.

Let us denote by $\mathscr{A}(\mathbf{z})$ the area between the zero-concentration curve and the concentration curve for income source z and by $\mathscr{B}(\mathbf{z})$ the area between the zero-concentration curve and the maximum-concentration curve.[16] We define the income-factor concentration index, $\mathscr{I}(\mathbf{z})$, corresponding to the distribution $\mathbf{z}$, as follows:

$$\mathscr{I}(\mathbf{z}) = \frac{\mathscr{A}(\mathbf{z})}{\mathscr{B}^{max}(\mathbf{z})}, \tag{11}$$

with $\mathbf{z} = \boldsymbol{\pi}, \mathbf{w}$.

This measure has considerable intuitive appeal: It is the area between the zero-concentration curve $\mathscr{L}^e(\mathbf{z}, p)$ and the concentration curve for income source $\mathscr{L}(\mathbf{z}, p)$, divided by the area between the zero-concentration curve $\mathscr{L}^e(\mathbf{z}, p)$ and the maximum-concentration curve $\mathscr{L}^{max}(\mathbf{z}, p)$.[17] This measure lies therefore between -1 (when individuals at the bottom own source z and individuals at the top own source z_-) and 1 (when individuals at the bottom own source z_- and individuals at the top own source z). It is equal to zero when the area of the concentration curve is the same as that of the zero-concentration curve.[18] Furthermore, note that $\mathscr{I}(\mathbf{z}) = -\mathscr{I}(\mathbf{z}_-)$ (see the Appendix for further details).

[16] Formally, $\mathscr{A}(\mathbf{z}) = \frac{1}{2n}\sum_{i=1}^{n}\left[\left(\mathscr{L}^e\left(\mathbf{z}, \frac{i}{n}\right) + \mathscr{L}^e\left(\mathbf{z}, \frac{i-1}{n}\right)\right) - \left(\mathscr{L}\left(\mathbf{z}, \frac{i}{n}\right) + \mathscr{L}\left(\mathbf{z}, \frac{i-1}{n}\right)\right)\right]$ and $\mathscr{B}^{max}(\mathbf{z}) = \left|\frac{1}{2n}\sum_{i=1}^{n}\left[\left(\mathscr{L}^e\left(\mathbf{z}, \frac{i}{n}\right) + \mathscr{L}^e\left(\mathbf{z}, \frac{i-1}{n}\right)\right) - \left(\mathscr{L}^{max}\left(\mathbf{z}, \frac{i}{n}\right) + \mathscr{L}^{max}\left(\mathbf{z}, \frac{i-1}{n}\right)\right)\right]\right|$, with $max = m, M$.

[17] Note that the areas between the curves $\mathscr{L}^M(\mathbf{z}, p)$ and $\mathscr{L}^e(\mathbf{z}, p)$ and the curves $\mathscr{L}^e(\mathbf{z}, p)$ and $\mathscr{L}^m(\mathbf{z}, p)$ are the same for specific functional form of $\mathscr{L}(\mathbf{y}, p)$ and for certain values of z (see the appendix for further details).

[18] The latter may happen without that the two curves coincide.

In light of the relationship previously discussed between the concentration curves and the ideal-typical social systems proposed by Milanovic, we can also interpret such indicator as a measure of the degree of capitalism of a given social system. Furthermore, the new type of capitalism can also be considered as a multiple sources of income society.
The metric proposed is not a rank-based measure of association between labor and capital (Atkinson and Lakner, 2017). Indeed, a monotone transformation in the marginal distributions would affect the index by changing the ranking in the distribution of total income.[19]
Although it may seem of little interest to consider negative values of the index, they have a powerful meaning in terms of income composition dynamics, as stated by the following Definition.

Definition 4.5. *Let $sign_{t,t+1}$ be the sign of $\mathscr{I}^t(\boldsymbol{z}) \cdot \mathscr{I}^{t+1}(\boldsymbol{z})$, where $\mathscr{I}^t(\boldsymbol{z})$ is the metric at time t, while $\mathscr{I}^{t+1}(\boldsymbol{z})$ the one at time $t+1$. We say that a change in the structure of income composition across the distribution of income occurs at time t if $sign_{t,t+1} < 0$.*

When a change in sign occurs at time $t+1$ (i.e., $sign_{t,t+1} < 0$), those individuals who mainly own source z at time t, earn mainly source z_- at time $t+1$ and vice versa. The normalization coefficient $\mathscr{B}^m(\mathbf{z})$ is a function of $\mathscr{L}(\mathbf{y}, p)$, z and p'', while the coefficient $\mathscr{B}^M(\mathbf{z})$ is a function of $\mathscr{L}(\mathbf{y}, p)$, z and p'. To simplify the notation, let us generally denote by $\mathscr{B}(\mathbf{z})$ the denominator of the metric. A more compact expression for the index is, for $z = \pi$, as follows:

$$\mathscr{I}(\boldsymbol{\pi}) = \frac{w\pi(\tilde{\mu}_w - \tilde{\mu}_\pi)}{\mathscr{B}(\boldsymbol{\pi})}, \tag{12}$$

where $\tilde{\mu}_\pi = \frac{1}{2n}\sum_{i=0}^{n}\left(\sum_{j=0}^{i}\alpha_j + \sum_{j=0}^{i+1}\alpha_j\right)$ and $\tilde{\mu}_w = \frac{1}{2n}\sum_{i=0}^{n}\left(\sum_{j=0}^{i}\beta_j + \sum_{j=0}^{i+1}\beta_j\right)$ are the areas of the concentration curves for labor and capital multiplied by $\frac{1}{w}$ and $\frac{1}{\pi}$ respectively.[20] Similarly, for $z = w$, we have:

$$\mathscr{I}(\mathbf{w}) = \frac{w\pi(\tilde{\mu}_\pi - \tilde{\mu}_w)}{\mathscr{B}(\mathbf{w})}. \tag{13}$$

Equations 12 and 13 simply mean to illustrate the functional forms of this indicator once we mainly focus on the concentration of capital and of labor at the top, respectively. Specifically, when equation 12 is positive, then the capital is concentrated primarily at

[19] For a full discussion on rank-based measures of association, see Dardanoni and Lambert (2001), Atkinson and Lakner (2017), Aaberge, Atkinson and Königs (2018).
[20] Note that one minus twice $\tilde{\mu}_z$ gives the pseudo-Gini of income source z (see Shorrocks, 1982).

the top of the income distribution and the labor at the bottom. Conversely, when equation 13 is positive, then the labor is concentrated primarily at the top of the income distribution and the capital at the bottom. As we have previously discussed, the following relationship holds true: $\mathscr{I}(\boldsymbol{\pi}) = -\mathscr{I}(\mathbf{w})$.

The two functions $\tilde{\mu}_\pi$ and $\tilde{\mu}_w$ have a precise dynamics: They increase (decrease) when the source in question moves towards the bottom (top) of the distribution. These areas can thus be considered as approximate metrics of the indicator previously introduced.[21] In a similar manner, the function $\tilde{\mu}_y$ is a measure of income inequality: When it rises so does the surface of the Lorenz curve, by therefore reducing its distance from the egalitarian line.

At a first glance, this indicator may bear resemblance to the *pseudo-Gini coefficient*, firstly proposed by Fei et al. (1978). However, these two metrics are very different from each other. Let us consider, for instance, the pseudo-Gini for capital income $\bar{\mathscr{G}}_\pi$, which can be written in the following way: $\bar{\mathscr{G}}_\pi = 1 - 2\tilde{\mu}_\pi$. This indicator is equal to zero when all individuals have the same *absolute level* of capital income, regardless of whether their total incomes may differ. Let me better illustrate this point with a simple example. Suppose we have a population of three individuals, whose relative income shares are described by the following vector $(y_1, y_2.y_3) = \left(\frac{1}{10}, \frac{3}{10}, \frac{6}{10}\right)$. The pseudo-Gini coefficient is equal to zero when the vector of the relative shares of capital income is of the following form $(\alpha_1, \alpha_2.\alpha_3) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. Now, given that individual 1 has the same share of capital income of individual 3, this makes the former individual more capital abundant than the latter individual. Therefore, in a society as such, an increase in the capital share of income would rise the income of individual 1 relatively more than the income of individual 3. For this reason, the pseudo-Gini coefficient cannot be regarded as a measure of the relationship between the functional and personal income distribution.

To conclude this section, it is of utmost importance to highlight that, just as there are many indices that measures income inequality, there can be many different ways to measure income composition inequality. This aspect lays the grounds for future methodological research on the matter.

[21] We can also observe that the term $\tilde{\mu}_\pi$ (and similarly $\tilde{\mu}_w$ and $\tilde{\mu}_y$) can be expressed as follows: $\tilde{\mu}_\pi = \sum_{i=1}^n \alpha_i \left(\frac{2n-2i+1}{2n}\right)$. It suffices to note that $\tilde{\mu}_\pi = \frac{1}{2n} \sum_{i=0}^n \left(\sum_{j=0}^i \alpha_j + \sum_{j=0}^{i+1} \alpha_j\right) = \frac{1}{2n} \sum_{i=1}^n \left(2 \sum_{j=1}^i \alpha_j + \alpha_i\right) = \frac{1}{n} \sum_{i=0}^n \sum_{j=0}^i \alpha_j + \frac{1}{2n} \sum_{i=0}^n \alpha_i$, from which we obtain the result.

4.5 From Functional to Personal Distribution of Income

In this Section, we further investigate the relationship between functional income distribution and income inequality, in light of the novel metric previously illustrated. To this end, let us consider the well-known relationship between $\tilde{\mu}_y$ (the area of the Lorenz curve) and the Gini coefficient:

$$\mathscr{G} = 1 - 2\tilde{\mu}_y. \tag{14}$$

The latter can be further developed, so to obtain:

$$\mathscr{G} = 1 - 2(z(\tilde{\mu}_z - \tilde{\mu}_{z_-}) + \tilde{\mu}_{z_-}). \tag{15}$$

The Gini coefficient can therefore be expressed as a function of the two approximate metrics of income-factor concentration $\tilde{\mu}_z$ and $\tilde{\mu}_{z_-}$ and of the factor share z. If we take the derivative of $\mathscr{G}$ with respect to z, we obtain:

$$\frac{\partial \mathscr{G}}{\partial z} = 2(\tilde{\mu}_{z_-} - \tilde{\mu}_z). \tag{16}$$

The elasticity of personal income Gini to changes in the factor shares is (two times) the difference between the areas of the two concentration curves. Note that when $\tilde{\mu}_{z_-} - \tilde{\mu}_z < 0$, then an increase in the capital share reduces income inequality.
If we consider the standard decomposition of total income Gini into inequality contributed by each income source:[22]

$$\mathscr{G} = z\mathscr{R}_z\mathscr{G}_z + z_-\mathscr{R}_{z_-}\mathscr{G}_{z_-}, \tag{17}$$

where $\mathscr{R}_z = \frac{cov(r(y),z)}{cov(r(z),z)}$ is the correlation ratio between the source z and total income, $r(y)$ and $r(z)$ are the individual's ranks according to total income and source z respectively and $\mathscr{G}_z$ is the Gini coefficient of income source z, we can write:

$$\frac{\partial \mathscr{G}}{\partial z} = \mathscr{R}_z\mathscr{G}_z - \mathscr{R}_{z_-}\mathscr{G}_{z_-}, \tag{18}$$

and by combining both equations 16 and 18 we get:

$$2\left(\tilde{\mu}_{z_-} - \tilde{\mu}_z\right) = \mathscr{R}_z\mathscr{G}_z - \mathscr{R}_{z_-}\mathscr{G}_{z_-}. \tag{19}$$

According to Milanovic, "for the rising share of capital income to increase overall income Gini, we need therefore to have two 'transmission' tools, Gini coefficient of capital income

[22] See Lerman and Yitzhaki (1985) for further details.

and $\mathscr{R}_\pi$, positive and high" (Milanovic, 2017), or, from a more technical perspective, the following condition must hold: $\mathscr{R}_\pi \mathscr{G}_\pi > \mathscr{R}_w \mathscr{G}_w$. It appears that such condition is well captured by the sign of the indicator previously introduced, which exclusively depends on $\tilde{\mu}_{z_-} - \tilde{\mu}_z$. Therefore, equation 16 shows that, for the analysis of the relationship between the functional and personal distribution of income, the indicator we propose can be considered as a tool capable of linking these two distributions. For example, if the capital share of income was rising, then income inequality would grow only if the income composition inequality was greater than zero.

5 The Case of a Two-Person Economy

Let us now consider the scenario in which the population is divided into two groups (i.e., $n = 2$) of equal size. This exercise is of interest for two main reasons. The first reason is that, due to the lack of data, it may be difficult sometimes to compute the index previously illustrated, which requires information concerning the composition of individuals' income for the entire population.[23] The second reason is that the $n = 2$ version of the index has some interesting mathematical properties that deserve to be exposed.

Let us denote by y_p the income of the bottom $p\%$ of the income distribution and by y_{1-p} the income of the top $(1-p)\%$, with $y_p \in]0, \frac{1}{2}[$. Figure 2 provides us with a graphical representation of the concentration curves for $n = 2$. The income-factor concentration index, $\mathscr{I}$, with $n = 2$ takes the following mathematical form:

$$\mathscr{I}_2(\mathbf{z}, p) = b_{z,p} w\pi(\eta_p^{z-} - \eta_p^z) = t_{z,p}\ \rho\ I(\mathbf{z}, p), \tag{20}$$

where $\rho = w\pi$, $I(\mathbf{z}, p) = \eta_p^{z-} - \eta_p^z$ and the normalization coefficient $b_{z,p}$ is defined as follows:

$$b_{z,p} = \begin{cases} \frac{1}{y_p z} & \text{if } \ y_p > \eta_p^z \\ \frac{1}{[\min(y_p, z) - y_p z]} & \text{if } \ y_p < \eta_p^z \end{cases}.$$

[23] This information is generally provided by the surveys, which, however, tend to underestimate the income of individuals at the top of the distribution.

Concentration Curves for Income Source with $n = 2$

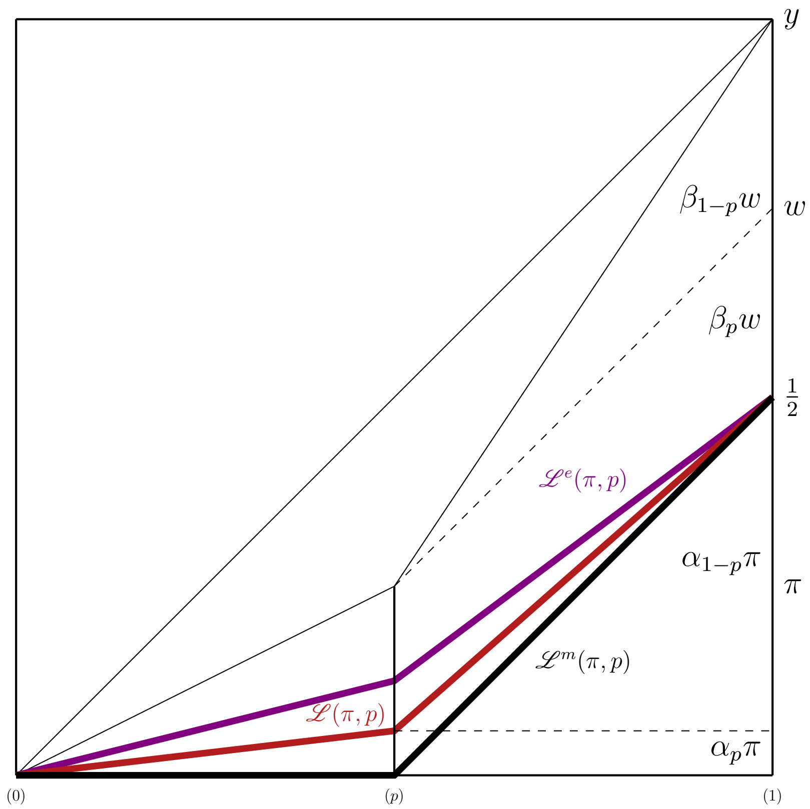


Figure 2: A graphical representation of the methodology in which two people (or groups) with different income ($y_p < y_{1-p}$, with $p = \frac{1}{2}$) and two sources of the same amount ($\pi = w$) are compared. The carnelian line $\mathscr{L}(\boldsymbol{\pi}, p)$ is the concentration curve for capital, the violet line $\mathscr{L}^e(p)$ is the zero-concentration curve, while the black line $\mathscr{L}^m(\pi, p)$ is the maximum-concentration curve. The following values have been here assigned: $y_p = 0.25$, $\pi = w = \frac{1}{2}$, $\alpha_p = 0.12$ and $\beta_p = 0.38$.

This version can thus be regarded as the product of three elements, notably $t_{z,p}$, ρ and $I(\mathbf{z}, p)$. An interesting way to grasp their meaning is to rewrite the index as follows:

$$\mathscr{I}_2(\mathbf{z}, p) = b_{z,p} \begin{vmatrix} w & 0 \\ 0 & \pi \end{vmatrix} \begin{vmatrix} \eta_p^{z-} & \eta_p^{z} \\ \eta_{1-p}^{z-} & \eta_{1-p}^{z} \end{vmatrix},$$

where the product of the two determinants ρ and $I(\mathbf{z}, p)$ is simply the determinant of the following matrix A:

$$A = \begin{pmatrix} \eta_p^{z-} w & \eta_p^{z}\pi \\ \eta_{1-p}^{z-} w & \eta_{1-p}^{z}\pi \end{pmatrix}.$$

The $\mathscr{I}_2(\mathbf{z}, p)$ index can hence be rewritten as the product between the determinants of two matrices and a normalizing coefficient. The first determinant, ρ, adjusts the degree of concentration for the level of income sources. The second determinant, $I(\mathbf{z}, p)$, is, instead, the channel through which the issue of income-source concentration is addressed. Interestingly, we can note that the following matrix A^*:

$$A^* = \begin{pmatrix} \beta_p & \alpha_p \\ \beta_{1-p} & \alpha_{1-p} \end{pmatrix},$$

whose determinant equals the component $I(\mathbf{z}, p)$, comes from the following relationship:

$$\bar{y} = A^* \bar{x},$$

where $\bar{y} = \begin{pmatrix} y_p \\ y_{1-p} \end{pmatrix}$ and $\bar{x} = \begin{pmatrix} w \\ \pi \end{pmatrix}$, which in turns is equivalent to the following system of equations:

$$\begin{cases} y_p = \beta_p w + \alpha_p \pi \\ y_{1-p} = \beta_{1-p} w + \alpha_{1-p} \pi \end{cases}.$$

When the matrix A^* is nonsingular (i.e., $\det A^* \neq 0$, thus $I(\mathbf{z}, p) \neq 0$), then we can write: $\bar{x} = (A^*)^{-1}\bar{y}$. It is of interest to observe that when $\det A^* = 1$, then ownerships of labor and capital are separated between individual 1 and 2. This explains why the coefficient $I(\mathbf{z}, p)$ can be considered as a proxy of $\mathscr{I}(\mathbf{z})$.

Another way of writing the $n = 2$ version of this indicator is the following. Assume that $y_1 < w$ and $\mathbf{z} = \boldsymbol{\pi}$, then we have:[24]

$$\mathscr{I}_2(\boldsymbol{\pi}, p) = 1 - \frac{\alpha_1}{y_1}. \tag{21}$$

Equation 21 illustrates that the level of income composition inequality is in this very case determined by the ratio $\frac{\alpha_1}{y_1}$. This ratio combines individual 1's endowment of capital and overall income. When the ratio is greater than one, then individual 1 is more capital poor than income poor. When it is equal to one, then she is as capital poor as income poor and when it is lower than one, then she is more income poor than capital poor.

[24] This is a plausible assumption: In the contrary case, the labor share of income would have been lower than the capital share. In fact, if y_1 was greater than w, given that $y_1 < \frac{1}{2}$ by assumption, than we would have $w < \pi$. The latter is not supported by the empirical evidences concerning the developed countries (Stockhammer, 2013), with the exception of Mexico (Negrete, 2015).

Therefore, income composition inequality is positive when the poorest part of the society is more capital poor than income poor and negative in the opposite case.
Let us now illustrate several properties of the $I(\mathbf{z}, p)$. First, the capital to labor ratio can be expressed as follows:

$$\frac{\pi}{w} = \frac{\frac{1}{1+\varphi} - \beta_{1-p}}{\frac{\varphi}{1+\varphi} - \alpha_p}, \tag{22}$$

where $\varphi = \frac{y_p}{y_{1-p}}$, from which we simply derive the following result.

Proposition 5.1. *A variation in φ has no effect on $\frac{\pi}{w}$ iff $I(\boldsymbol{z}, p) = 0$. Formally:*

$$\frac{\partial \frac{\pi}{w}}{\partial \varphi} = 0 \iff I(\boldsymbol{z}, p) = 0. \tag{23}$$

Proposition 23 sheds light on the relationship between income inequality, as measured by the ratio φ and factor shares of income $(\frac{\pi}{w})$.[25] A variation in φ does not affect the ratio $\frac{\pi}{w}$ when the determinant of the matrix A^* is equal to zero.
Let us now consider the relationship between the determinant $I(\mathbf{z}, p)$ and the between-group Gini coefficient G.[26] Precisely:

$$\frac{\partial G}{\partial z} = I(\boldsymbol{z}_{-}, p)\ p. \tag{24}$$

An increase in the factor share z reduces the between-group inequality G according to the degree of income-source concentration and the share of poor people, p. If we let p be equal to $\frac{1}{2}$ (thus we divide the population into two groups of equal size) and if we set $z = \pi$, then we get:

$$\frac{\partial G}{\partial \pi} = \frac{\alpha_{\frac{1}{2}} - \beta_{\frac{1}{2}}}{2}. \tag{25}$$

Equation 25 bears resemblance with equation 16. Specifically, in a two-person economy the condition for the rising share of capital income to increase income Gini is $I(\mathbf{z}, p) > 0$, or $\det A^* > 0$.

6 Empirical Application

In this Section, we apply the method previously illustrated to the case of six European economies, namely Finland, France, Germany, Italy, Norway and The Netherlands.

[25] It is easy to note that $\frac{\partial \frac{\pi}{w}}{\partial \varphi} > 0$ when $I(\mathbf{z}, p) > 0$. In particular, when $I(\mathbf{z}, p) = 1$, an increase in φ raises the ratio $\frac{\pi}{w}$ of the same amount. Indeed, when $I(\mathbf{z}, p) = 1$ then $y_p = \pi$ and $y_{1-p} = w$, thus $\frac{\pi}{w} = \frac{y_p}{y_{1-p}}$.
[26] See the appendix for further details.

The data used come from the European Union Statistics of Income and Living Conditions (EU-SILC), which provide a representative sample of the European population. This data are firstly produced by the national statistical offices and later harmonized and released by Eurostat. In our analysis, we consider the period between 2007 and 2016. The country samples vary between 7000 and 19000 units and the unit of analysis is the household.

Our analysis relies on specific definitions of capital and labor incomes.[27] Precisely, we define capital income as the sum of household income from rental of a property or land, interests, dividends, capital from capital investments in unincorporated business, the capital component of gross cash benefits or losses from self-employment (including royalties) and pensions from individual private plans. The capital component of self-employment income, which is not directly provided by EU-SILC, is imputed by means of the procedure proposed by Glyn (2011). We attribute the average payroll income of the entire sample to represent the labor income component of the self-employed and "the margin of value added per head [...] is then regarded as accruing to the [self-employer] as property income" (Glyn, 2011, p. 8).[28] As stated by Glyn, none of the methods adopted by the literature to decompose the self-employment income into its labor and capital components is wholly unproblematic. In this respect, also our method presents some issues. Namely, by considering the economy average payroll income as a threshold to determine the capital and labor components of self-employment income, we risk to underestimate the capital component for those sectors in which the sectorial average payroll income is lower than the economy average payroll income, and viceversa. However, we believe this decomposition to be more sophisticated than that which automatically attributes two thirds of the self-employment income in its labor component, and one third in its capital component. Furthermore, note that to the best of our knowledge, this is the first time such method is applied to decompose self-employment income at

[27]The definitions of capital and labor income can be, to a certain extent, arbitrary. For instance, Cirillo, Corsi and D'Ippoliti (2017), who investigate the dynamics of the functional and personal distributions of income at the European level before and after the crisis, and who also rely on EU-SILC data, provide a slightly different definition of capital and labor income from that we propose. Indeed, their definition of income does not include self-employment remuneration.

[28]Precisely, we firstly compute the country average payroll income per household, $\mu_{payroll}$. Then, we decompose self-employment income into its capital and labor components in the following way. Let us denote by y_{se} the income from self-employment provided by EU-SILC. We can write that $y_{se} = y_{se}^{\pi} + y_{se}^{w}$, where $y_{se}^{w} = \begin{cases} y_{se} & \text{if } y_{se} \leq \mu_{payroll} \\ \mu_{payroll} & \text{if } y_{se} > \mu_{payroll} \end{cases}$, while $y_{se}^{\pi} = \begin{cases} 0 & \text{if } y_{se} \leq \mu_{payroll} \\ y_{se} - \mu_{payroll} & \text{if } y_{se} > \mu_{payroll} \end{cases}$.

the micro level (as this method is generally adopted to decompose macroeconomic variables). Labor income is defined as the difference between total household gross income minus capital income.[29]
To overcame the issue of negative values, we replace the bottom part of the concentration curves for which such problem occurs with the horizontal line (i.e., the x-axis).[30]
Let us begin the analysis with some descriptive statistics. Table 1 presents the income shares of four different income groups, defined as follows: *0-50%*, *50-90%*, *90-95%* and *95-100%*. These shares are computed for the six countries in 2007 and 2016, with the exception of Italy (2007 and 2015) due to missing information.

Table 1: Income Shares

Income Group	**Norway**		**Germany**		**Netherlands**		**Italy**		**France**		**Finland**	
	2007	*2016*	*2007*	*2016*	*2007*	*2016*	*2007*	*2015*	*2007*	*2016*	*2007*	*2016*
0-50%	24%	24%	23%	23%	26%	22%	18%	19%	24%	24%	22%	23%
50-90%	50%	49%	48%	48%	47%	47%	48%	48%	48%	46%	49%	48%
90-95%	9%	9%	10%	10%	9%	10%	11%	11%	10%	9%	10%	10%
95-100%	15%	17%	17%	17%	16%	19%	19%	19%	16%	18%	17%	17%
Gini	0.36	0.37	0.38	0.39	0.34	0.39	0.45	0.45	0.37	0.38	0.39	0.39
IFC	0.51	0.59	0.48	0.50	0.54	0.63	0.53	0.49	0.38	0.46	0.46	0.45

The distribution of total income is almost the same in 2007 and 2016 for all countries, with the sole exception of The Netherlands, where the 3% of total income has moved from the bottom 50% to the top 5%, by hampering the Gini coefficient of 0.5 percentage points. Italy displays the higher level of total income inequality, with a Gini coefficient above 0.4, and a $\frac{top\ 5\%}{bottom\ 50\%}$ ratio lower than 1 in 2007, and equal to 1 in 2015.
Tables 2 and 3, instead, show the distributions of the capital and labor shares respectively, with individuals being indexed by their income ranking. Following Shorrocks (1982) and Atkinson and Lakner (2018), we may call these shares as "pseudo-shares". Let us take a closer look at Table 2. A simple way to read this Table is the following:

[29] The sources of labor income that we consider are: gross employee cash or near cash income, gross non-cash employee income, employers' social insurance contributions, value of goods produced for own consumption, unemployment benefits, old-age benefits, survivor' benefits, sickness benefits, disability benefits, education-related allowances, family/children related allowances, social exclusion not elsewhere classified, housing allowances, regular inter-household cash transfers received and income received by people aged under 16.

[30] When a given variable at stake displays negative values, the bottom part of the corresponding concentration curve lies below the horizontal axe.

in 2007 in Norway the individuals at the bottom 50% of the total income distribution earn the 13% of total capital income in the economy. The share of the capital income earned by the same income group in 2016 is 4% points lower than the share in 2007.

Table 2: Capital Shares (Income Ranking)

Income Group	**Norway**		**Germany**		**Netherlands**		**Italy**		**France**		**Finland**	
	2007	*2016*	*2007*	*2016*	*2007*	*2016*	*2007*	*2015*	*2007*	*2016*	*2007*	*2016*
0-50%	13%	9%	11%	9%	11%	6%	5%	7%	14%	15%	10%	9%
50-90%	22%	20%	25%	30%	22%	23%	30%	33%	29%	29%	31%	36%
90-95%	7%	9%	9%	8%	8%	9%	13%	15%	12%	12%	12%	13%
95-100%	57%	61%	53%	52%	57%	60%	49%	44%	43%	46%	45%	39%
Capital Share	6%	8%	9%	6%	7%	7%	17%	16%	8%	12%	13%	10%
Gini	0.36	0.37	0.38	0.39	0.34	0.39	0.45	0.45	0.37	0.38	0.39	0.39
IFC	0.51	0.59	0.48	0.50	0.54	0.63	0.53	0.49	0.38	0.46	0.46	0.45

The first thing to notice is that in Norway, Germany and The Netherlands the individuals at the top 5% of the income distribution earn more than 50% of all capital income in the economy. On the contrary, in Italy, France and Finland the share of capital income earned by the same income group is lower than 50%. Moreover, differently from the total income distribution, the capital income distribution has changed over the period considered in almost all countries. However, it is hard to identify clear patterns between the six countries.

In Norway and The Netherlands the capital income has moved from the bottom 50% to the top 5% between 2007 and 2016. A similar scenario can be described for France, although less markedly than in the previous two countries. In Italy and Finland, instead, the top 5% has seen a reduction of its capital income share. However, while in Italy the capital income has shifted towards the bottom 50% and the middle class ($50 - 90\%$), in Finland it has moved towards the middle class only. Finally, in Germany the middle class has gained capital income from both the top 10% and the bottom 50%.

Table 3: Labor Shares (Income Ranking)

Income Group	Norway		Germany		Netherlands		Italy		France		Finland	
	2007	*2016*	*2007*	*2016*	*2007*	*2016*	*2007*	*2015*	*2007*	*2016*	*2007*	*2016*
0-50%	25%	25%	25%	24%	27%	24%	21%	21%	25%	26%	24%	24%
50-90%	51%	51%	51%	50%	49%	49%	53%	52%	50%	49%	51%	50%
90-95%	9%	9%	10%	10%	9%	10%	10%	10%	9%	9%	9%	10%
95-100%	12%	12%	13%	14%	13%	15%	14%	15%	14%	14%	13%	14%
Labor Share	94%	92%	91%	94%	93%	93%	83%	84%	92%	88%	87%	90%
Gini	0.36	0.37	0.38	0.39	0.34	0.39	0.45	0.45	0.37	0.38	0.39	0.39
IFC	0.51	0.59	0.48	0.50	0.54	0.63	0.53	0.49	0.38	0.46	0.46	0.45

Table 3, instead, shows that very little changes have been registered in the labor income distribution for almost all countries between 2007 and 2016. It is only worth mentioning that in The Netherlands the labor income has mainly moved from the bottom 50% to the top 10%. Furthermore, in each country the middle class earns, on average, the 50% of all the labor income in the economy.

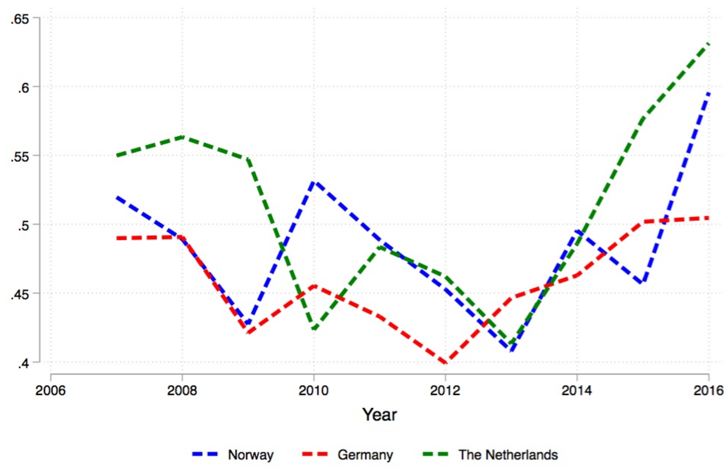
As shown in Table 1, the dynamics of the four total income shares is relatively well captured by that of the Gini coefficient. However, what can we say about the joint dynamics of the capital and labor shares? Are the capital and labor incomes better distributed across the populations or, rather, more concentrated at the top and at the bottom of the income distributions? Do these countries bear more resemblance to a classical capitalism, characterised by a rich "capitalist class" and a poor "working class", or rather to a new capitalism, where all individuals earn multiple sources of income? To answer these questions, we apply the method previously developed.

Figures 3 shows the overall dynamics of income composition inequality for the two groups of European countries, respectively. The first group is composed by Germany, Norway and The Netherlands, whilst the second by Italy, France and Finland. To begin with, note that the IFC index ranges between 0.4 and 0.6 in all countries. However, the two groups seem to follow different trends in terms of income composition over the time considered. Indeed, income composition inequality in the first group follows a U-shaped pattern, with its major peaks in 2007 and 2016, and its lowest peak in 2013-2014 (a).

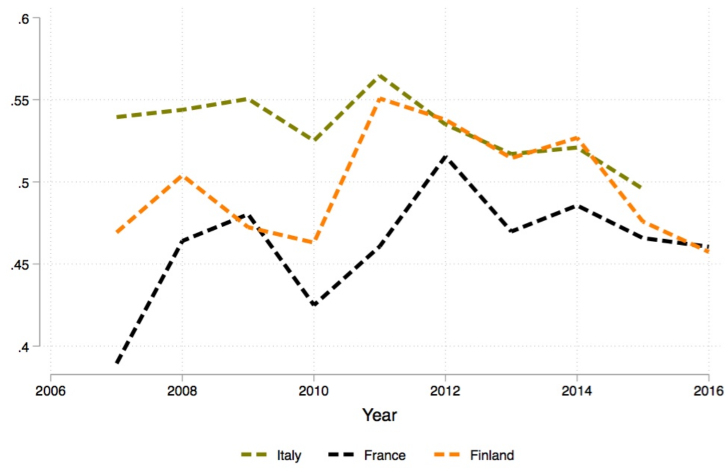
On the contrary, income composition inequality follows an inverted U-shaped curve in the second group, with its peaks in 2011/2012 and its lowest levels at the beginning and at the end of the period considered (b).

The patterns of Germany and The Netherlands almost coincide in both levels and trends,

Income Composition Inequality



(a)

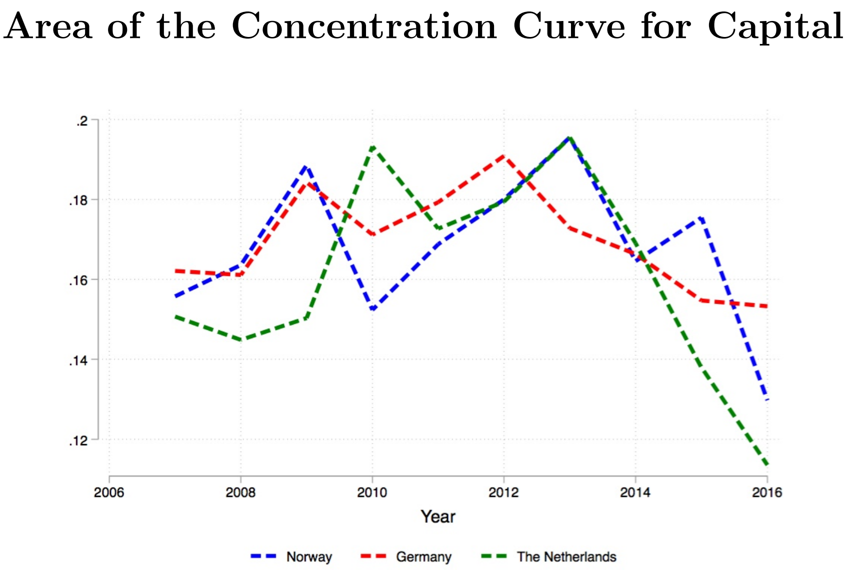


(b)

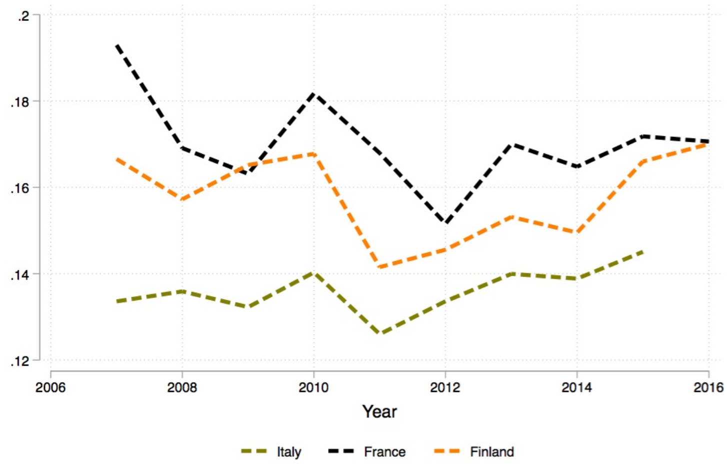
Figure 3: The series of income composition inequality 2007-2016, as measured by the income-factor concentration index. **Source**: Author's computation on basis of EU-SILC.

whilst that of Norway starts from a lower level in 2007, and ends with a higher level of income composition inequality in 2016.

All the countries of the second group exhibit a similar dynamic of income composition inequality. However, France and Italy show higher levels of the IFC index as compared to Finland. Following the framework previously discussed, we can say that the first three countries considered are moving towards becoming a classical capitalism, characterized



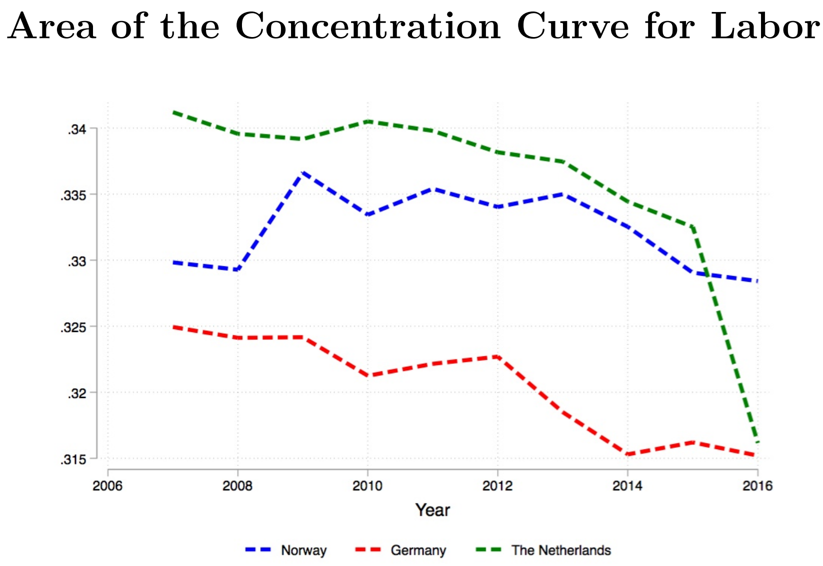
(a)



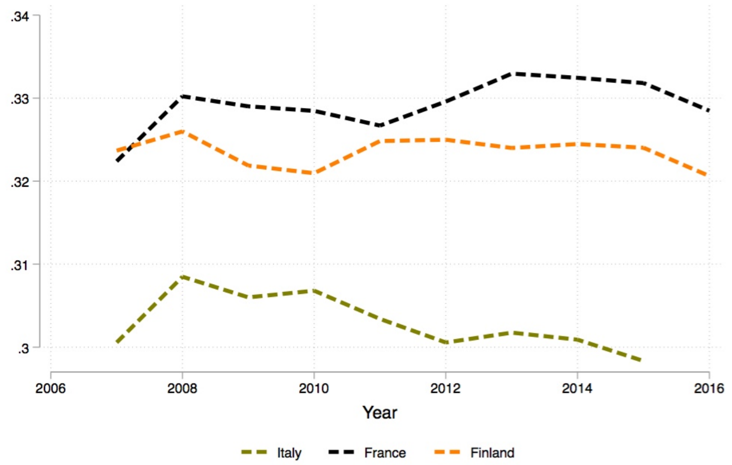
(b)

Figure 4: The series of the area of the concentration curves for capital income 2007-2016. **Source**: Author's computation on basis of EU-SILC.

by a group of rich people owning capital income and a group of poor people owning labor income. This type of society allows for a greater transmission of changes in the functional distribution of income into personal income inequality. Conversely, the second group of countries is moving towards becoming a new capitalism, in which both sources of income are better distributed across the entire population. In the latter society, the relationship between functional and personal distribution of income is relatively weak,



(a)



(b)

Figure 5: The series of the area of the concentration curves for labor income 2007-2016. **Source**: Author's computation on basis of EU-SILC.

implying that fluctuations in both the capital and labor shares of income have a less severe impact on the dynamics of income inequality.

At this point of the analysis, let us analyze the role played by the two components of the iIFC index, notably $\tilde{\mu}_w$ and $\tilde{\mu}_\pi$, in shaping its overall dynamics. The evolution of the areas of the concentration curves for capital and labor are illustrated by Figures 4 and 5. As already illustrated by Tables 2 and 3, the two metrics $\tilde{\mu}_w$ and $\tilde{\mu}_\pi$ follow completely independent patterns. Let us begin with the first group. For all countries the area of the concentration curve for capital rises up to 2013 and falls afterwords (Figure 4). We remind that an increase (decrease) in $\tilde{\mu}_\pi$ implies that the capital income moves towards the bottom (top) of the income distribution. Therefore, we can state that Germany, Norway and The Netherlands saw their capital income flowing in the hands of the bottom part of the income distribution up to 2013 and afterwords coming back into possession of the rich part of the population. At the same time, the almost flat motion of the area of the concentration curve for labor $\tilde{\mu}_w$ for all the countries (Figure 5) clearly suggests that the principal driver of income composition inequality was the fluctuation in capital income. A different story can be told with regards to the second group of countries. The evolution of income composition inequality for Finland, France and Italy has been characterized by the the capital income moving firstly towards the top (up to 2013) and then towards the bottom of the income distribution (from 2013 onwards).

6.1 Discussion

The objective of the previous section was to illustrate how the methodology developed in the first part of the paper can be applied to study the evolution of the income composition in different countries and across time. The empirical application has clearly revealed the extent to which the IFC index summarizes information on the joint concentration of capital and labor income across the income distribution, similarly to the way the Gini coefficient summarizes information on the distribution of income across the population. Furthermore, it has shown how the results can be interpreted in terms of *(i)* the evolution of the relationship between the functional and personal distribution of income and *(ii)* the dynamics of socio-economic systems as defined by Milanovic (2017). Having said that, a sound assessment of the underlying forces behind the trends previously presented would go well beyond the scope of this article, whose main focus was primarily to introduce the novel concept of income composition inequality, together with a statistical indicator for its measurement, the income-factor concentration (IFC) index. However, the study of income composition inequality through the IFC index raises a number of questions for future inquiry. Let us focus on two in particular.

From a *development perspective*, looking at the evolution of income composition inequality in a given country, jointly with its economic growth, is crucial to answer several, fundamental questions: does income composition inequality increase, or decrease as an economy grows? In other words, is there any relationship between the type of capitalism at work in a country and its economic development? If this is the case, is this relationship country-specific, or rather specific to the time period we are analyzing? And how does it relate to Kuznets' (1955) hypothesis on the relationship between income inequality and growth, and to Milanovic's (2016) theorization of the Kuznets Waves? To answer all these questions in a context in which survey data is scarce, the IFC index can be approximated by its $n = 2$ version. The latter version requires data on two representative individuals only: a rich and a poor representative individual. The rich individual can be represented by a given top share of the income distribution (e.g. top 10% or top 1%), whilst the poor individual by a given bottom share (e.g. 90% or 50%). As we have illustrated in the paper, the IFC index for $n = 2$ can be expressed as a function of two variables only: the relative share of capital (or labor) income and the relative share of total income of one of the two individuals only. This allows to make reasonable assumptions concerning the degree of the IFC index in the distant past.

From a *policy perspective*, it is important to understand the impact that redistribution policies have not only on income inequality, but also on income composition inequality. A tax and transfer scheme that mainly redistribute labor income (such as the one proposed by Kakwani, 1993) has the double effect of reducing income inequality and increasing income composition inequality. The latter would happen if we assumed that the pre-tax and transfer level of income composition inequality was positive (which is a reasonable assumption given the previous empirical applications). In a context where the capital income share is rising, a similar tax and transfer scheme may lead, in the long run, to an increase in income inequality via the resulting higher level of income composition inequality. Hence, studying the impact that a tax and transfer scheme has on a country's level of income composition inequality may help us highlighting the contradictory nature of current redistribution policies that, on the one hand, reduce income inequality in the short run, and on the other hand, increases income inequality in the long run via the increase in income composition inequality in a context of rising capital income share.

These examples illustrate the potential macroeconomic, as well as policy implications that the analysis of a country's income composition inequality can have, and lay the foundations for future research on the matter.

7 Conclusion

One of the most important findings from Piketty's *Capital in the Twenty-First Century* (Piketty, 2014) is the rise in the capital share of income in many developed countries over the last decades (see also Piketty, 2015). Similar results were also found by Stockhammer (2013), who shows that the labour shares has fallen over the past 25 years in the OECD countries. The dynamics of the capital share of income (and, hence, of the labor share) is influenced by many macroeconomic phenomena, such as technical change, globalization, financialisation, bargaining power and market power of firms, among others (see Stockhammer et al. 2018). The rise in the capital share of income is generally considered to be one of the causes that led to the increase in personal income inequality (Piketty, 2014; Bengtsson and Waldenstrom, 2018). However, the study of the link between changes in the capital share of income and changes in personal income inequality needs to be further investigated. For this reason, the present paper proposed a methodology to examine the relationship between the functional and personal distribution of income. To this end, it introduced the concept of inequality in income composition. If we decompose total income into *two* factors, such as capital and labour income, then income composition inequality is the extent to which the income composition is distributed unevenly across the income distribution. Inequality in income composition is *maximal* when individuals at the top and at the bottom of the income distribution separately earn the two different types of income. On the contrary, it is *minimal* when each individual earns the same composition of the two factors. Under a high level of income composition inequality the link between the functional and personal distribution of income is strong, whereas under a low level of income composition inequality the link is weak. We then constructed a summary statistic to measure income composition inequality, the income-factor concentration index. We showed that this summary statistic can be looked at in two ways. Firstly, from a technical perspective, it can be considered an elasticity of personal income inequality to fluctuations in the functional income distribution. In other words, it mathematically links the functional and personal distribution of income. Secondly, from a political economy perspective, it measures the "degree of capitalism" of a given social system. We then applied the methodology to study the evolution of income composition inequality in six European economies. Although these countries are characterized by different trends, they all display a positive value of the IFC index, indicating that capital incomes are mainly concentrated at the top of the income distribution, whereas labor incomes are mainly concentrating at the bottom. Finally, we discussed how the study of income composition inequality can pave the way for further research on different economic aspects, from development to public finance.

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A Sign of the Indicator

In order to show that $\mathscr{I}(\mathbf{z}) = -\mathscr{I}(\mathbf{z}_{-})$, we need to proof that $\mathscr{B}^m(\mathbf{z}) = \mathscr{B}^M(\mathbf{z}_{-})$. The latter relationship states that the denominator of $\mathscr{I}(\mathbf{z})$ equals that of $\mathscr{I}(\mathbf{z}_{-})$. To this end, we should consider two different maximum-concentration curves.

From equations 12 and 13 we can simply notice that $\mathscr{A}(\mathbf{z}) = -\mathscr{A}(\mathbf{z}_{-})$, thus that the numerator of the indicator $\mathscr{I}$ changes its sign (and not its absolute level) according to the source we analyze. Without loss of generality, if we assume that source z is mainly concentrated at the top, and that source z_{-} at the bottom, then the relationship $\mathscr{B}^m(\mathbf{z}) = \mathscr{B}^M(\mathbf{z}_{-})$ can be written as follows:

$$\int_0^1 z\mathscr{L}(\mathbf{y},p)dp - \int_{p^*}^1 \left[\mathscr{L}(\mathbf{z},p) - z_{-}\right] dp = \int_0^{p^{**}} \mathscr{L}(\mathbf{y},p)dp + (1-p^{**})z_{-} - \int_0^1 z_{-}\mathscr{L}(\mathbf{y},p)dp.$$

Considering that p^* and p^{**} are such that $\mathscr{L}(\mathbf{y},p^*) = \mathscr{L}(\mathbf{y},p^{**}) = z_{-}$, then $p^* = p^{**}$, and the relationship holds true.

B Normalization Coefficient

As stated before in this paper, we show that for specific functional forms of the Lorenz curve for income $\mathscr{L}(\mathbf{y},p)$, and for specific values of z (and, thus, z_{-}), the following relationship holds true:

$$\mathscr{L}^M(\mathbf{z}) - \mathscr{L}^e(\mathbf{z}) = \mathscr{L}^e(\mathbf{z}) - \mathscr{L}^m(\mathbf{z}). \tag{26}$$

For simplicity, let us move to the continuous space. Suppose, therefore, that we have three continuous distribution functions: $\mathbf{y}$, $\boldsymbol{\pi}$, $\mathbf{w}$. The relationship 26 is equivalent to the following one:

$$z\int_0^1 \mathscr{L}(\mathbf{y},p)dp - \int_{p''}^1 \left(\mathscr{L}(\mathbf{y},p) - z_{-}\right) dp = \int_0^{p'} \mathscr{L}(\mathbf{y},p)dp + \left(1-p'\right) z - z\int_0^1 \mathscr{L}(\mathbf{y},p)dp. \tag{27}$$

We remember that p' s.t. $\mathscr{L}(\mathbf{y},p') = z$, p'' s.t. $\mathscr{L}(\mathbf{y},p\grave{\imath}'') = 1 - z$ and $\mathbf{z} = \pi, w$. From equation 27 we can write:

$$2z\int_0^1 \mathscr{L}(\mathbf{y},p)dp = \int_{p''}^1 \left(\mathscr{L}(\mathbf{y},p) - z_{-}\right) dp + \int_0^{p'} \mathscr{L}(\mathbf{y},p)dp + \left(1-p'\right) z.$$

If we call $p^{'} = f(z)$ and $p^{''} = f(z_{-})$, where $f(y) = \mathscr{L}^{-1}(\mathbf{y}, p)$ then, after further manipulations, we get:

$$\int_0^1 \mathscr{L}(\mathbf{y}, p)dp = 1 + \frac{1}{z - z_{-}} \int_{f(z)}^{f(z_{-})} \mathscr{L}(\mathbf{y}, p)dp + \frac{zf(z) - z_{-}f(z_{-})}{z - z_{-}},$$

which is true only if the following relationship is satisfied:

$$(z - z_{-}) \int_0^1 f(y)dy = \int_z^{z_{-}} f(y)dy. \tag{28}$$

Note that equation 28 is true for $\pi = w$, $\pi = 1$, $w = 1$, regardless of the functional form of $\mathscr{L}$, and for the family of functions f of the form $f(x) = x^n$, for $n = 1, +\infty, -\infty$ only.

C Result 5.1

Provided that $y_p = \alpha_p \pi + \beta_p w$, and $y_{1-p} = \alpha_{1-p}\pi + \beta_{1-p}w$, where $y_p + y_{1-p} = y = \pi + w$, we can write:

$$\begin{aligned}
\varphi &= \frac{\beta_p w + \alpha_p \pi}{\beta_{1-p}w + \alpha_{1-p}\pi}, && \iff \\
y_p(\beta_{1-p}w + \alpha_{1-p}\pi) &= y_{1-p}(\beta_p w + \alpha_p \pi), && \iff \\
\frac{\pi}{w} &= \frac{\beta_p y_{1-p} - \beta_{1-p} y_p}{\alpha_{1-p} y_p - \alpha_p y_{1-p}}, && \iff \\
\frac{\pi}{w} &= \frac{\beta_p - \varphi \beta_{1-p}}{-\alpha_p + \varphi \alpha_{1-p}}, && \iff \\
\frac{\pi}{w} &= \frac{1 - (1-\varphi)\beta_{1-p}}{\varphi - (1-\varphi)\alpha_p}, && \iff \\
\frac{\pi}{w} &= \frac{\frac{1}{1-\varphi} - \beta_{1-p}}{\frac{\varphi}{1+\varphi} - \alpha_p}.
\end{aligned}$$

If we now take the first derivative of $\frac{\pi}{w}$ with respect to φ and we further manipulate, we obtain result 5.1.

D Relationship between Gini and IFC for $n = 2$

Let us rewrite y_1 (from equation 1) as follows:

$$y_1 = \beta_1 w \pm \alpha_1 w + \alpha_1 \pi.$$

After some algebraic manipulations, we get:

$$y_1 = Iw + \alpha_1,$$

where I is the distribution's component of the index $\mathscr{I}$ when $n = 2$. Let us now recall the expression of the Gini coefficient:

$$G = 1 - \sum_{k=1}^{n} (x_{k+1} - x_k)(y_{k+1} + y_k),$$

where the whole population is divided into n groups, and x_k, y_k represent the bottom x_k% of the population, and their cumulative income respectively. When $n = 2$ we can write:

$$G = 1 - py_p - (1 - p)y_p,$$

where p is the share of the poor group, and $(1 - p)$ the share of the rich. The following can be derived:

$$G = 1 - p(Iw + \alpha_p) - (1 - p)$$

whence:[31]

$$G = p(\alpha_{1-p} - Iw)$$

from which, by taking the derivative with respect to w, we obtain equation 24.

[31] It can be noticed that $G = p(\alpha_{1-p} - Iw) = p(1 - y_p) = py_{1-p}$, which is a different way to express the two-groups Gini coefficient. Indeed, it clearly appears from the equation that inequality rises when either the share of poor people increases, or when the income share of the rich augments.