Inequality Analysis Tools

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Based on:

My papers:

- "Deprivation and Social Exclusion" (joint with W. Bossert and V. Peragine), *Economica*, 74, 777-803, 2007.
- "Dynamic Measures of Individual Deprivation" (joint with W. Bossert), *Social Choice and Welfare*, 28, 77-88, 2007.

On some notes downloaded from the web (thanks to colleagues for making them available!)

And on:

Atkinson, A.B. and A. Brandolini:
http://siteresources.worldbank.org/INTDECINEQ/Resources/1149208-1169141694589/Global_World_Inequality.pdf

Chakravarty, S.R.: “Relative Deprivation and Satisfaction Orderings”, Keio Economic Studies, 34, 17-31, 1997.

Duclos, J-Y., J.M. Esteban and D. Ray, “Polarization: Concepts, Measurement, Estimation,” Econometrica, 72, 1737-1772, 2004.

Esteban. J.M. and D. Ray, “On the Measurement of Polarization,” Econometrica, 62, 819-851, 1994.

Hey, J.D. and P. Lambert: “Relative Deprivation and the Gini Coefficient: Comment”, Quarterly Journal of Economics, 95, 567-573, 1980.

Podder, N., "Relative Deprivation, Envy and Economic Inequality," Kyklos, 3, 353-376, 1996.

Yitzhaki, S. (1979): “Relative Deprivation and the Gini Coefficient”, Quarterly Journal of Economics, 93, 321-324, 1979.

Notation

Income distribution:

$\mathbb{N}$ denotes the set of all positive integers and $\mathbb{R}$ ($\mathbb{R}_+$, $\mathbb{R}_{++}$) is the set of all (all non-negative, all positive) real numbers.

An income distribution is a list incomes of different individuals.

If there are n persons in the society, the incomes could be listed as $x_1, x_2, ..., x_n$ where $x_i \geq 0$ is the income of person i, with (the strict inequality) $>$ for at least one i, $1 \leq i \leq n$, and n is an arbitrary positive integer. We write $x = (x_1, x_2, ..., x_n)$. Let $\mathbb{D}$ be the space of all such distributions.

We write $\lambda(x)$ (or simply λ) for the mean of x and $m(x)$ (or simply m) for the median of x

$\overline{x}$ represents the illfare ranked permutation of x, that is $\overline{x}_1 \leq \overline{x}_2 \leq ...\overline{x}_n$.

Notation

The distinct levels of incomes are collected in a vector $(x_1, ..., x_k)$ where $k \leq n$. Let π_j indicate the population share composed of individuals experiencing the same level of income, x_j. A distribution is $(\pi, x) \equiv (\pi_1, ..., \pi_k; x_1, ..., x_k)$, $x_i \neq x_j$ for all $i, j \in \{1, ..., k\}$. Let Ω be the space of all distributions. $\overline{x}$ indicates the illfare ranked permutation of the vector x.

Notation

Functioning failures distribution:

The distinct levels of functioning failures are collected in a vector $(q_1, ..., q_k)$ where $k \leq n$.. Let π_j indicate the population share composed of individuals with the same level of functioning failures, q_j. A distribution is $(\pi, q) \equiv (\pi_1, ..., \pi_k; q_1, ..., q_k)$, $q_i \neq q_j$ for all $i, j \in \{1, ..., k\}$. Let Θ be the space of all distributions. $\overline{q}$ indicates the illfare ranked permutation of the vector q.

Inequality Measures

Definition

An inequality measure is a function I from D to R which, for each distribution x in D indicates the level I(x) of inequality in the distribution.

Four Basic Properties

Definition

We say that x is obtained from y by a *permutation of incomes* if x = Py, where P is a permutation matrix.

Ex

$$x = Py = \begin{bmatrix} 010 \\ 100 \\ 001 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 6 \end{bmatrix}$$

Symmetry (Anonymity)

If x is obtained from y by a permutation of incomes, then I(x)=I(y).

All differences across people have been accounted for in x

Def

We say that x is obtained from y by a *replication* if the incomes in x are simply the incomes in y repeated a finite number of times

Ex

$$x = (y_1, y_1, y_2, y_2, \ldots\ldots, y_n, y_n)$$

$$x = (6, 6, 6, 1, 1, 1, 8, 8, 8)$$

Replication Invariance (Population Principle)

If x is obtained from y by a replication, then I(x)=I(y).

Can compare across different sized populations

Def

We say that x is obtained from y by a *proportional change* if x=αy, for some α > 0.

Ex

$$y = (6, 1, 8) \qquad x = (12, 2, 16)$$

Scale Invariance (Zero-Degree Homogeneity)

If x is obtained from y by a proportional change, then I(x)=I(y).

Relative inequality

Def

We say that x is obtained from y by a *(Pigou-Dalton) regressive transfer* if for some i, j:

i) $y_i \leq y_j$

ii) $y_i - x_i = x_j - y_j > 0$

iii) $x_k = y_k$ for all k different to i,j

Ex

$$y = (2, 6, 7) \qquad x = (1, 6, 8)$$

Transfer Principle

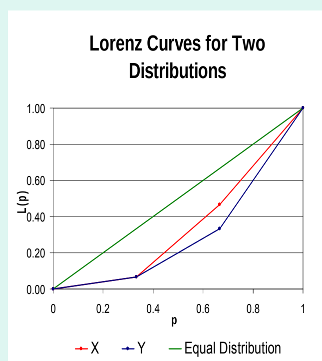
If x is obtained from y by a regressive transfer, then I(x) > I(y).

The Lorenz Curve and the Four Axioms

Symmetry and *Replication invariance* satisfied since permutations and replications leave the curve unchanged.

Proportional changes in incomes do not affect the LC, since it is normalized by the mean income. Only *shares* matter. So it is *scale invariant*.

A regressive transfer will move the Lorenz curve further away from the diagonal. So it satisfies *transfer principle*.



$$y = (1, 5, 9)$$
$$x = (1, 6, 8)$$

Lorenz Consistency

Def

An inequality measure *I: D→R* is *Lorenz consistent* whenever the following hold for any *x* and *y* in D: (i) if x Lorenz dominates y, then I(x) < I(y), and (ii) if x has the same Lorenz curve as y, then I(x) = I(y).

Theorem

An inequality measure *I(x)* is Lorenz consistent if and only if it satisfies symmetry, replication invariance, scale invariance and the transfer principle.

Note

If Lorenz curves don't cross, then all relative measures follow the Lorenz curve.

If Lorenz curves cross, then some relative measure of inequality might be used to make the comparison. But the judgment may depend on the chosen measure.

Thinking about inequality

Amiel and Cowell, 1999, CUP

In each of the first nine questions you are asked to compare two distributions of income. Please state which of them you consider to be the **more unequally** distributed by circling A or B. If you consider that both of the distributions have the same inequality then circle both A and B.

1) A = (5, 8, 10) B = (10, 16, 20)

2) A = (5, 8, 10) B = (10, 13, 15)

3) A = (5, 8, 10) B = (5, 5, 8, 8, 10, 10)

4) A = (1, 4, 7, 10, 13) B = (1, 5, 6, 10, 13)

Inequality and proportionate and absolute income differences (% responses) (N=1108)

Numerical problems (q. 2)

		Add 5 units		
		Down	Up	Same
Double income	Down	8	2	5
	Up	15	3	17
	Same	37	5	9

Verbal questions (q. 11)

		Add 5 units		
		Down	Up	Same
Double income	Down	7	1	4
	Up	21	2	17
	Same	30	3	14

The effect on inequality of cloning the distributions (% responses) (N=1108)

	Numerical (q. 3)	Verbal (q. 12)
Down	31	22
Up	10	9
Same	58	66

The transfer principle (% responses) (N=1108)

	Numerical (q. 4)	Verbal (q. 13)
Agree	35	60
Strongly disagree	42	24
Disagree	22	14

Agree=A is more unequal than B
Strongly Disagree=B is more unequal than A
Disagree=A and B have the same inequality

What happens when we depart from scale invariance?

GLOBAL WORLD INEQUALITY: ABSOLUTE, RELATIVE OR INTERMEDIATE?

Anthony B. Atkinson
and
Andrea Brandolini

Aim

This paper examines how the conclusions on the evolution of world income inequality might be affected by abandoning the relative inequality criterion.

In particular:

- examine **methodological issues** and discuss classes of measures that combine the relative and absolute criterion.
- present the **results** from applying these different measures to the distribution of income in the **world**.
  - first discuss **international inequality**;
  - then give illustrative results on **global inequality.**

In particular:

- examine **methodological issues** and discuss classes of measures that combine the relative and absolute criterion.
- present the **results** from applying these different measures to the distribution of income in the **world**.
  - first discuss **international inequality**;
  - then give illustrative results on **global inequality.**

"global" differs from "international" in that within-country inequality is accounted for.

Question:

How shall we distribute/take a given sum of money within/from the population so that income inequality remains unchanged?

The answer social scientists generally give is:

“income inequality remains unchanged when all incomes are increased/decreased by the same **proportion**”.

They believe in **scale invariance**.

Inequality indices, *I*, are **relative**.

The answer social scientists generally give is:

"income inequality remains unchanged when all incomes are increased/decreased by the same **proportion**"

$I(10, 20, 30) = I(5, 10, 15) = I(20, 40, 60)$

$I(x) = I(cx)$ for all $c>0$, homogeneity of degree zero.

They believe in **scale invariance**.

Inequality indices, I, are **relative**.

Are social scientists correct?

It depends.

Other answers can be given to the same question.

Alternatives:

"Income inequality remains unchanged when all incomes are increased/decreased by the same **absolute** amount".

They believe in **translation invariance**.

Inequality indices, *I*, used are **absolute**.

Alternatives:

"Income inequality remains unchanged when all incomes are increased/decreased by the same **absolute** amount".

$I(10, 20, 30) = I(0, 10, 20) = I(15, 25, 35)$

$I(x) = I(x+t1^n)$ for all $t>0$.

They believe in **translation invariance**.

Inequality indices, *I*, used are **absolute**.

Alternatives:

"Income inequality remains unchanged when some kind of **combination** between an equal-proportion and an equal absolute amount increase/decrease of all incomes is performed".

They take a middle stand between the rightist view and the leftist view, and believe that an equal-proportion distribution increases inequality, while an equal-absolute amount distribution decreases inequality ("compromise property").

Inequality indices, I, used are **intermediate**.

They take a middle stand between the rightist view and the leftist view, and believe that an equal-proportion distribution increases inequality, while an equal-absolute amount distribution decreases inequality ("compromise property").

Inequality indices, *I*, used are **intermediate**.

The invariance condition of Bossert and Pfingsten (1990) is:

$I(x) = I(a[x+\xi 1^n]-\xi 1^n)$ for all $a>1$, where $\xi>0$ is a parameter indicating the inequality concept, value judgment parameter.

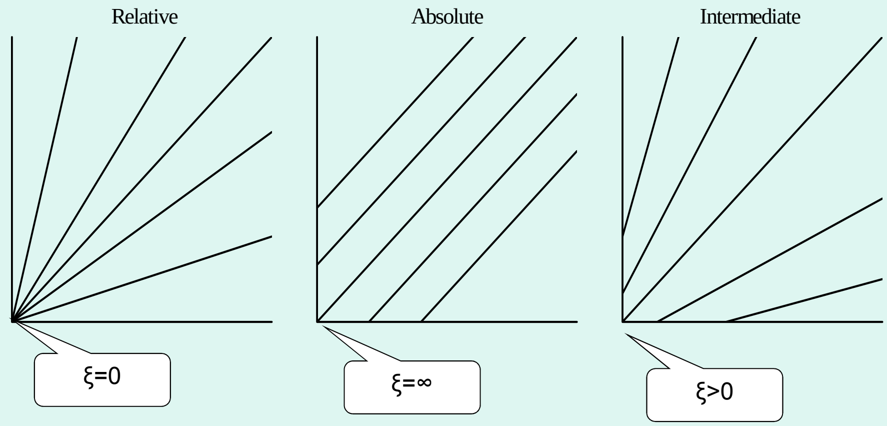
similar to Kolm's (1976) invariance condition

$sI(x) = I(s[x+m1^n]-m1^n])$ for all $s>0$, where $m>0$ is a parameter indicating the inequality concept, value judgment parameter.

What is ξ of Bossert and Pfingsten?

ξ is a parameter indicating the inequality concept, value judgment parameter, absolute value of origin of rays.

ISO-INEQUALITY CONTOURS FOR DIFFERENT INDEPENDENCE CRITERIA



There is no single correct answer to the distribution/taxation question posted above, the aforementioned views reflect value judgment in measuring income inequality.

In order to obtain reasonable inequality rankings, it may be desirable for different views of value judgment to be consulted in assessing income inequality.

Caveat: the inequality value of a population remains unchanged when incomes are measured in different currency units only for relative measures.

Results

Relative indices: the mean logarithmic deviation, the Gini index and the Theil index.

Absolute indices: absolute Gini index and the Kolm index for different values of its parameter.

Intermediate indices: Kolm, and Bossert and Pfingsten for different values of its parameters.

International income inequality

It examines the “international” rather than the “global” distribution of income since they study differences across countries in **per capita GDP** weighing each observation by the country’s **population**, but making no allowance for the distribution of income within the country.

Use real per capita GDP and population size for all countries and years in the period 1970-2000 for which both variables are available from the Penn World Table, Version 6.1 (Heston, Summers and Aten, 2002).

Use *real* incomes expressed in U.S. constant dollars.

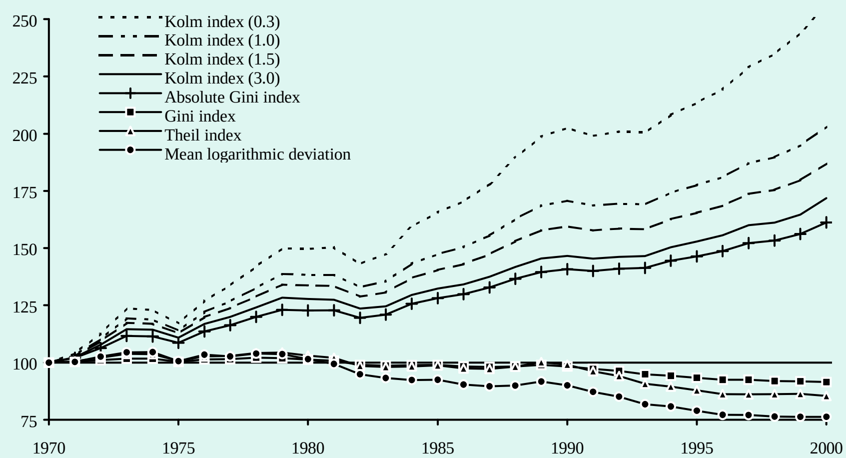
Full sample comprises 152 countries, but not all countries have a continuous run of data from 1970 to 2000: there are 30 or 31 observations for 106 countries, between 21 and 29 for another 27, and 15 or less for the remaining 29.

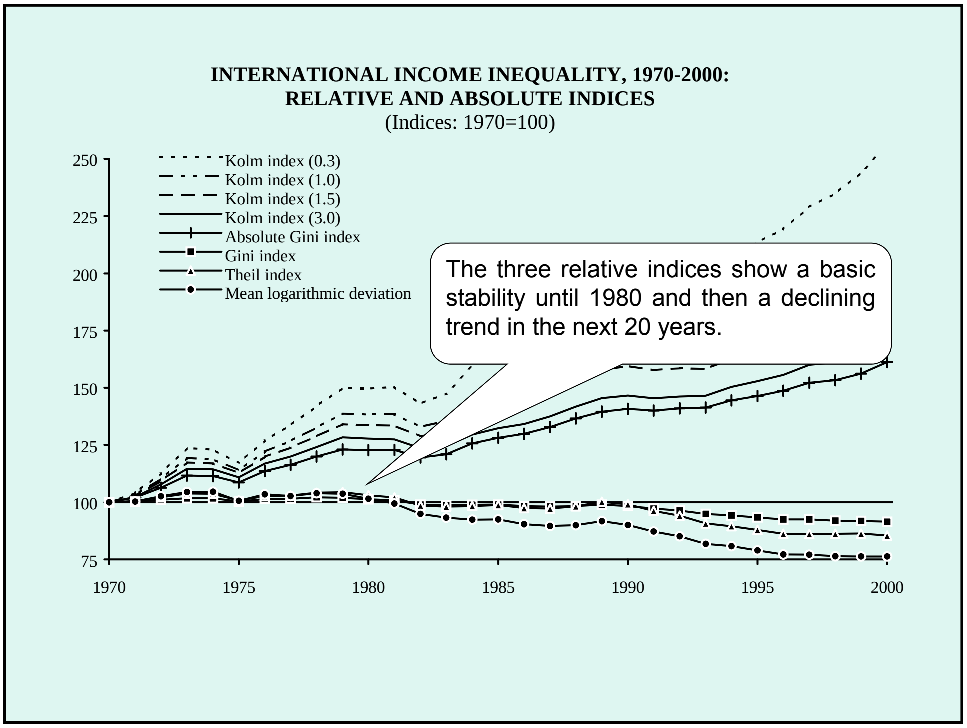
To avoid that measured trends reflect changes in country coverage, they concentrate on the sub-sample composed of the **106 countries** with 30 or 31 observations.

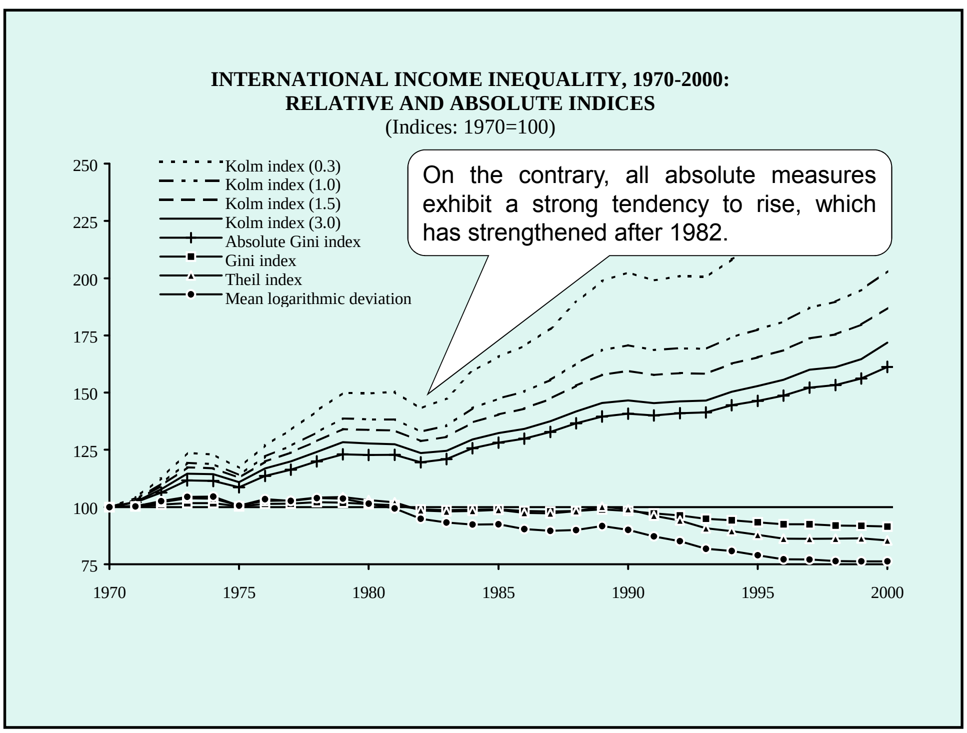
It includes **27** of the 30 countries which are currently member of the **OECD** (the Czech Republic, Poland and the Slovak Republic being those excluded), and all the **most populous nations but** for Russia and Vietnam (i.e. China, India, Indonesia, Brazil, Pakistan, Nigeria, Philippines, Thailand, Iran, Egypt, Ethiopia).

INTERNATIONAL INCOME INEQUALITY, 1970-2000: RELATIVE AND ABSOLUTE INDICES

(Indices: 1970=100)




INTERNATIONAL INCOME INEQUALITY, 1970-2000:
RELATIVE AND ABSOLUTE INDICES
(Indices: 1970=100)
Kolm index (0.3)
Kolm index (1.0)
Kolm index (1.5)
Kolm index (3.0)
Absolute Gini index
Gini index
Theil index
Mean logarithmic deviation
The three relative indices show a basic stability until 1980 and then a declining trend in the next 20 years.
250
225
200
175
150
125
100
75
1970
1975
1980
1985
1990
1995
2000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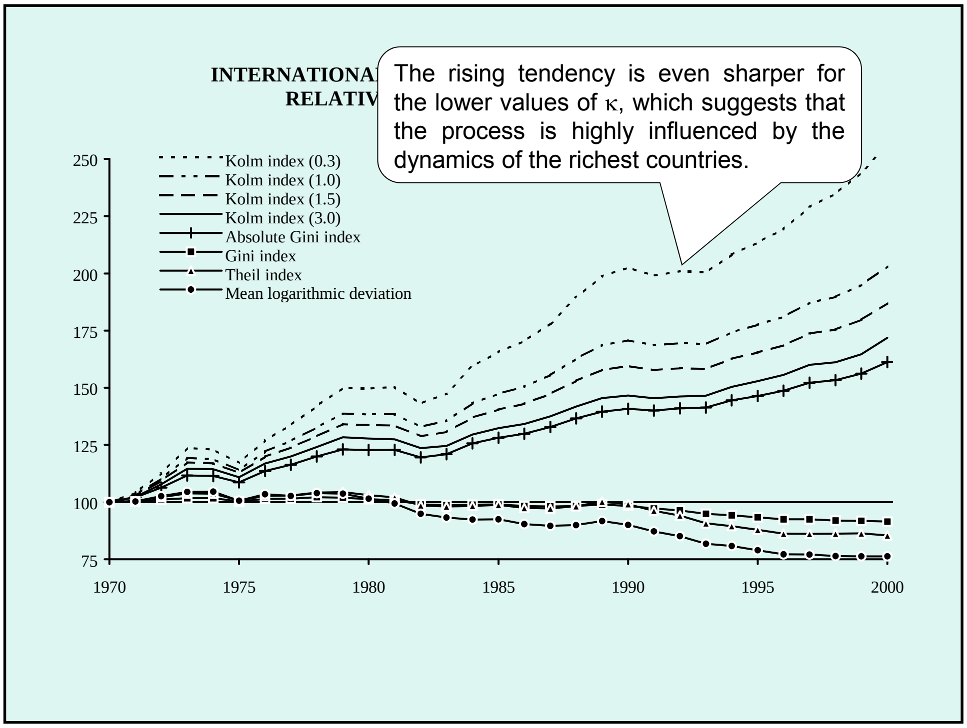
Kolm index (3.0)
Absolute Gini index
Gini index
Theil index
Mean logarithmic deviation
On the contrary, all absolute measures exhibit a strong tendency to rise, which has strengthened after 1982.
250
225
200
175
150
125
100
75
1970
1975
1980
1985
1990
1995
2000


INTERNATIONAL
RELATIV
The rising tendency is even sharper for the lower values of κ, which suggests that the process is highly influenced by the dynamics of the richest countries.
Kolm index (0.3)
Kolm index (1.0)
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Absolute Gini index
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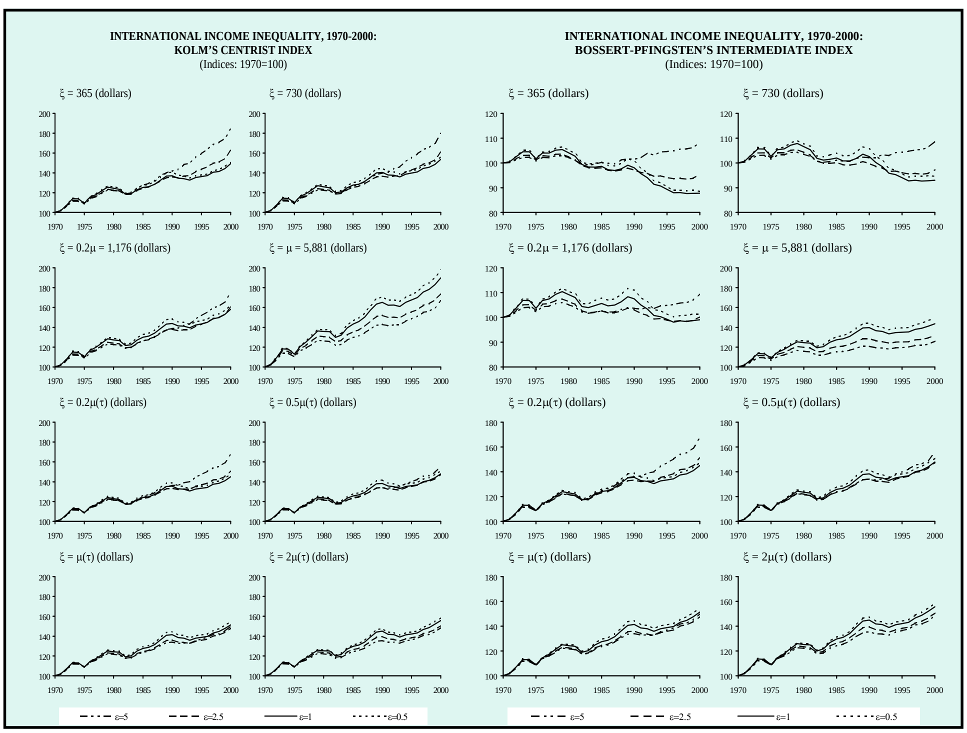
INTERNATIONAL INCOME INEQUALITY, 1970-2000:
KOLM'S CENTRIST INDEX
(Indices: 1970=100)
ξ = 365 (dollars)
ξ = 730 (dollars)
ξ = 0.2μ = 1,176 (dollars)
ξ = μ = 5,881 (dollars)
ξ = 0.2μ(τ) (dollars)
ξ = 0.5μ(τ) (dollars)
ξ = μ(τ) (dollars)
ξ = 2μ(τ) (dollars)
ε=5
ε=2.5
ε=1
ε=0.5
INTERNATIONAL INCOME INEQUALITY, 1970-2000:
BOSSERT-PFINGSTEN'S INTERMEDIATE INDEX
(Indices: 1970=100)
ξ = 365 (dollars)
ξ = 730 (dollars)
ξ = 0.2μ = 1,176 (dollars)
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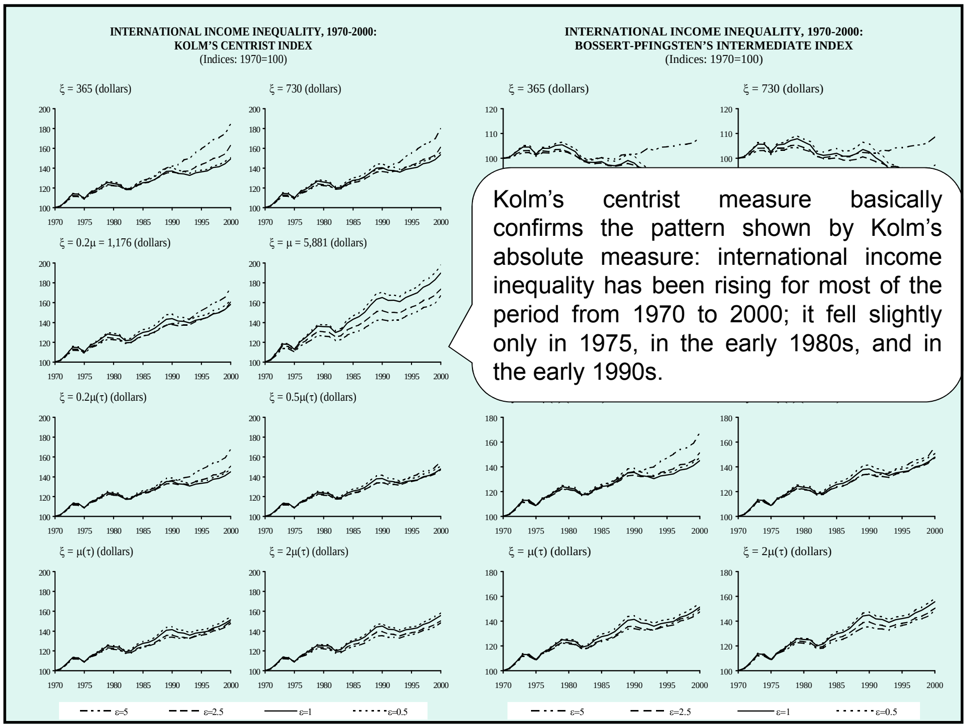
ξ = 0.5μ(τ) (dollars)
ξ = μ(τ) (dollars)
ξ = 2μ(τ) (dollars)
ε=5
ε=2.5
ε=1
ε=0.5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INTERNATIONAL INCOME INEQUALITY, 1970-2000:
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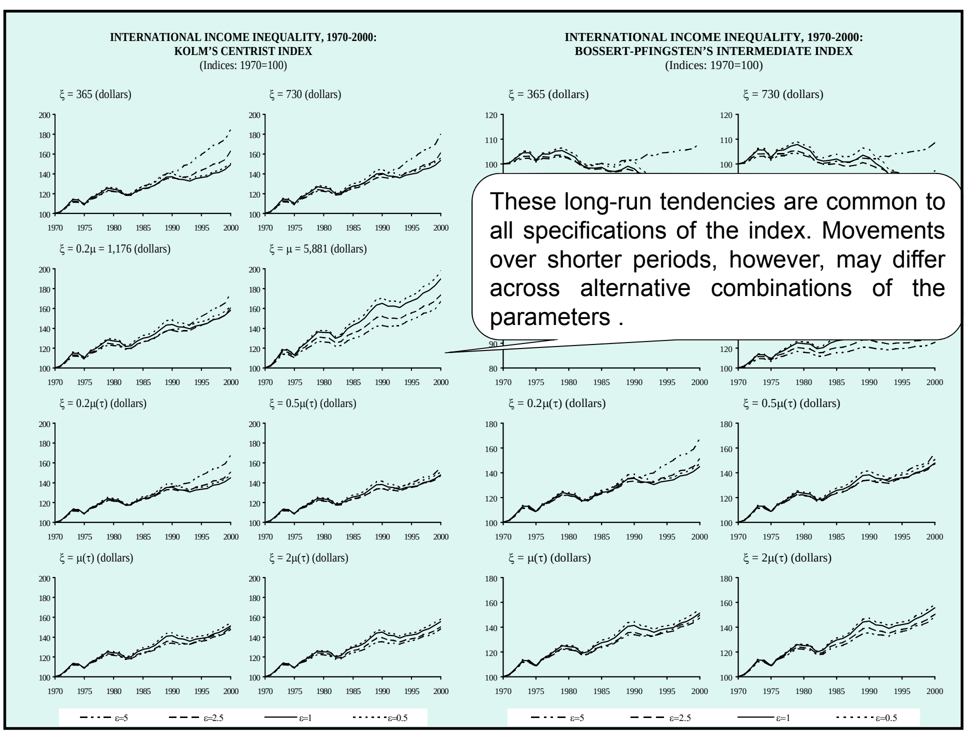
(Indices: 1970=100)
Kolm's centrist measure basically confirms the pattern shown by Kolm's absolute measure: international income inequality has been rising for most of the period from 1970 to 2000; it fell slightly only in 1975, in the early 1980s, and in the early 1990s.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INTERNATIONAL INCOME INEQUALITY, 1970-2000:
BOSSERT-PFINGSTEN'S INTERMEDIATE INDEX
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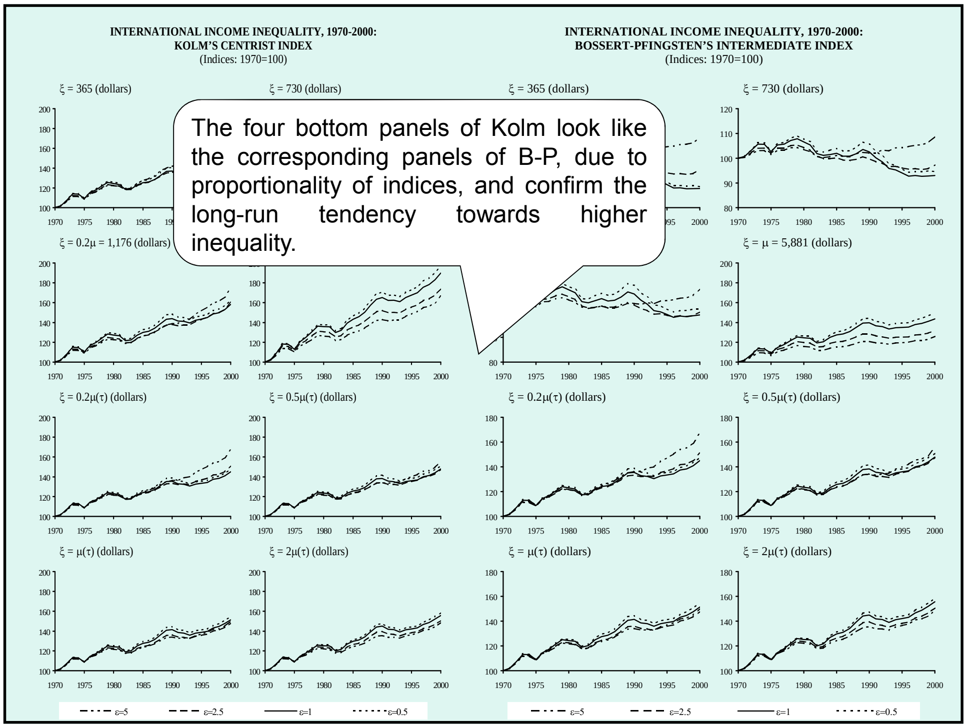
These long-run tendencies are common to all specifications of the index. Movements over shorter periods, however, may differ across alternative combinations of the parameters .



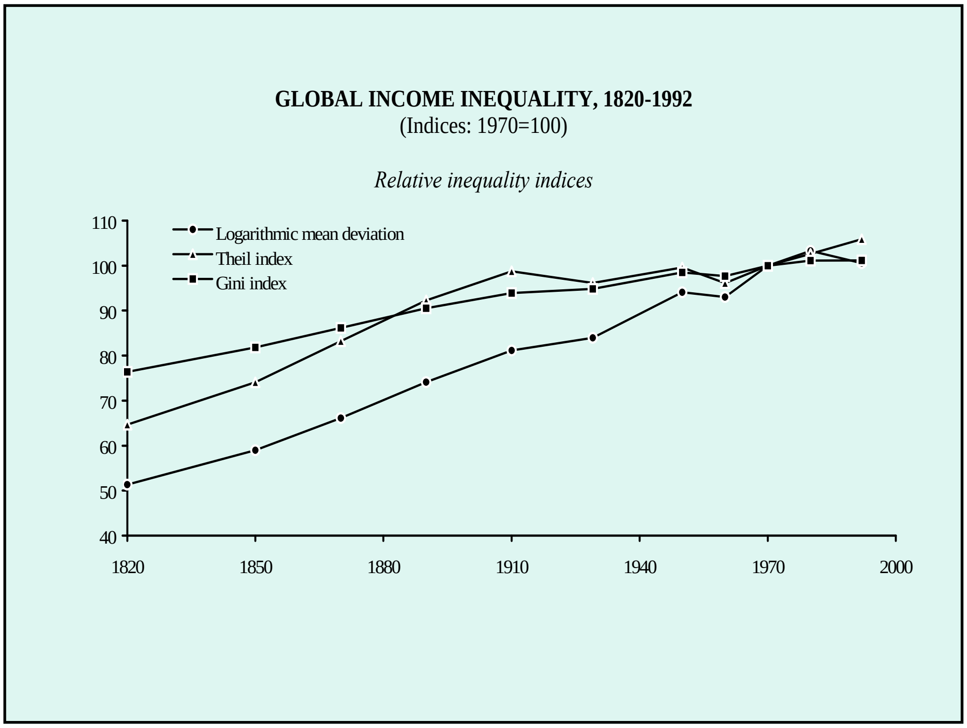
Global income inequality

A-B try to bring in **within-country inequality**.

The data for the world distribution of income are those constructed by **Bourguignon and Morrisson** (2002).

Their method is to use evidence on the national distribution (or the distribution for a grouping of countries) about the **income shares of decile groups, and the top 5 per cent**. The groups are treated as homogeneous, which means that the degree of overall inequality is under-stated, but their data provide a valuable starting point.

The distributional data are then combined with estimates of **national GDP per head**, expressed in constant purchasing power parity dollars (at 1990 prices), which are in turn derived from the historical time series constructed by **Maddison** (1995).


GLOBAL INCOME INEQUALITY, 1820-1992
(Indices: 1970=100)
Relative inequality indices
Logarithmic mean deviation
Theil index
Gini index
110
100
90
80
70
60
50
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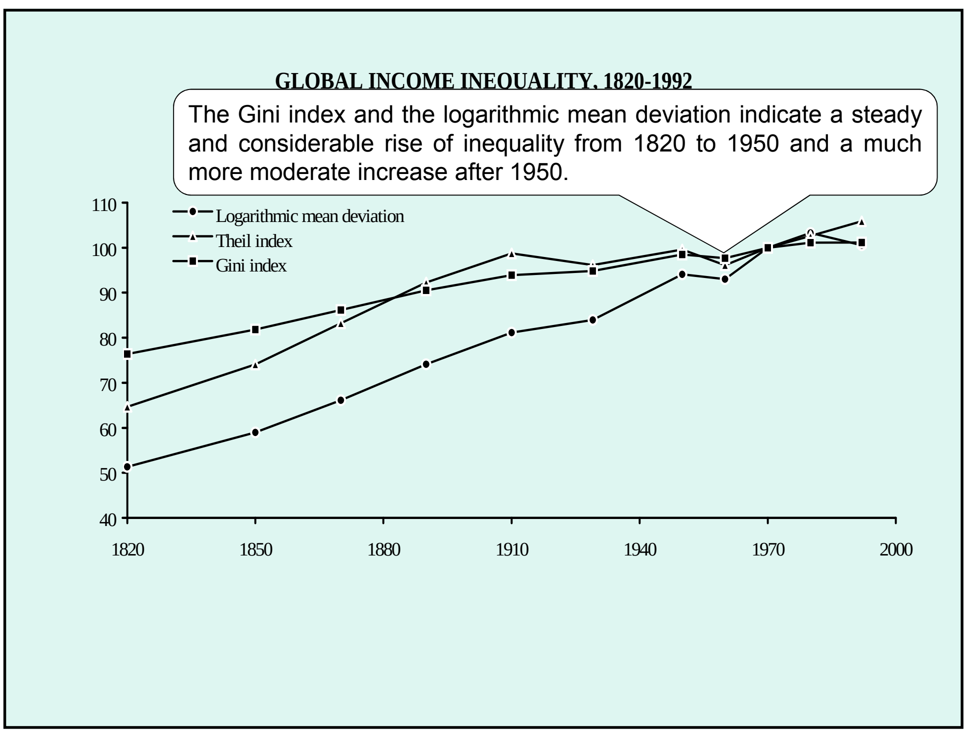
1820
1850
1880
1910
1940
1970
2000


GLOBAL INCOME INEQUALITY, 1820-1992
The Gini index and the logarithmic mean deviation indicate a steady and considerable rise of inequality from 1820 to 1950 and a much more moderate increase after 1950.
Logarithmic mean deviation
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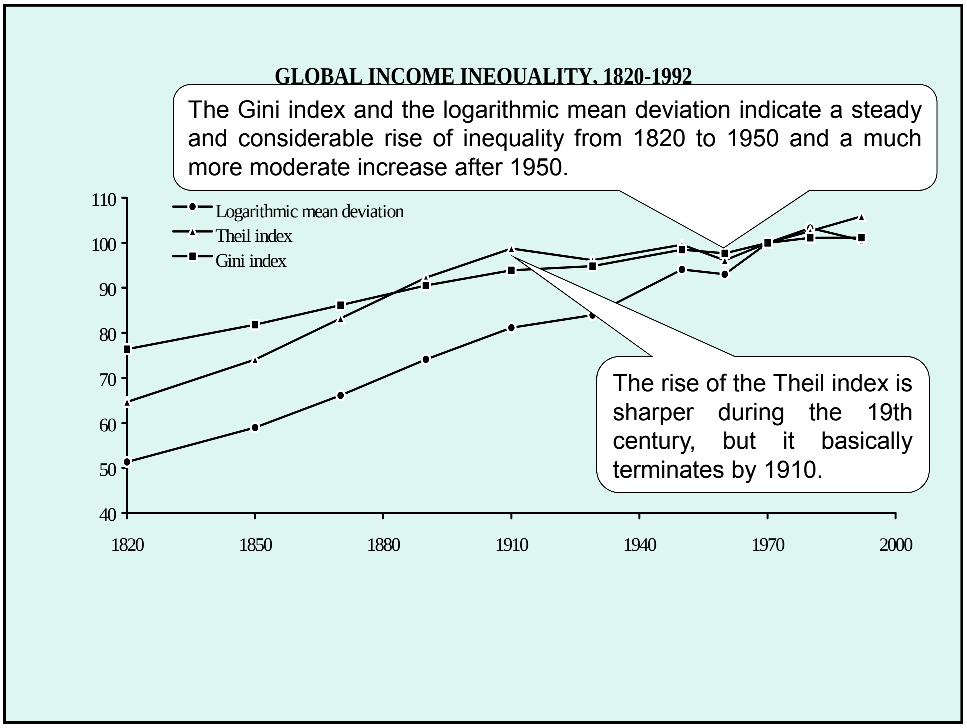
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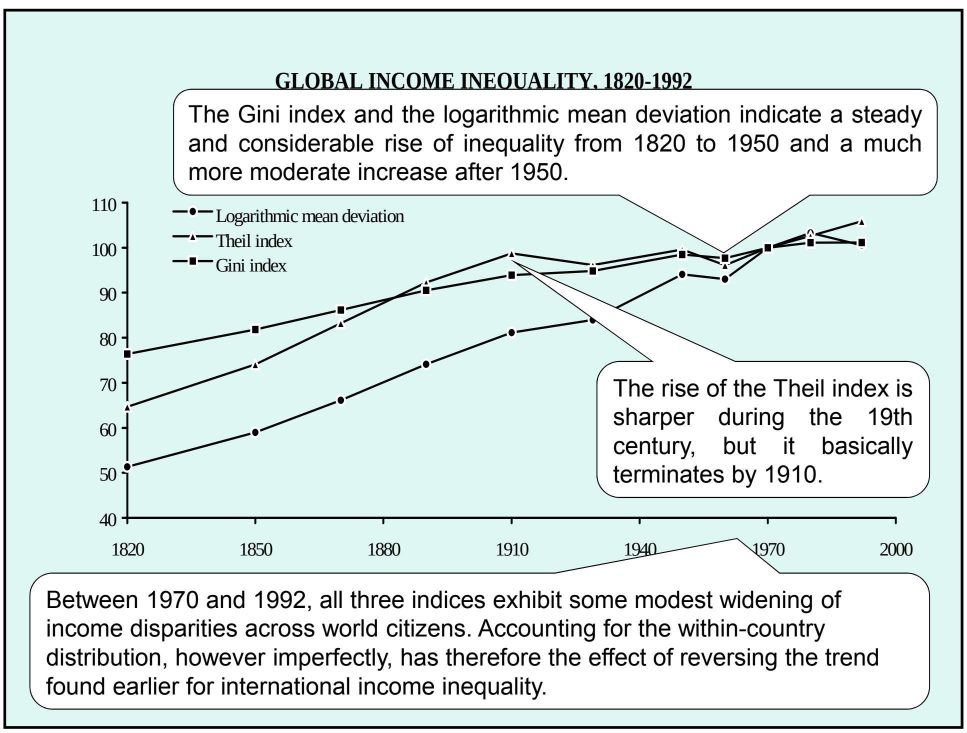
The rise of the Theil index is sharper during the 19th century, but it basically terminates by 1910.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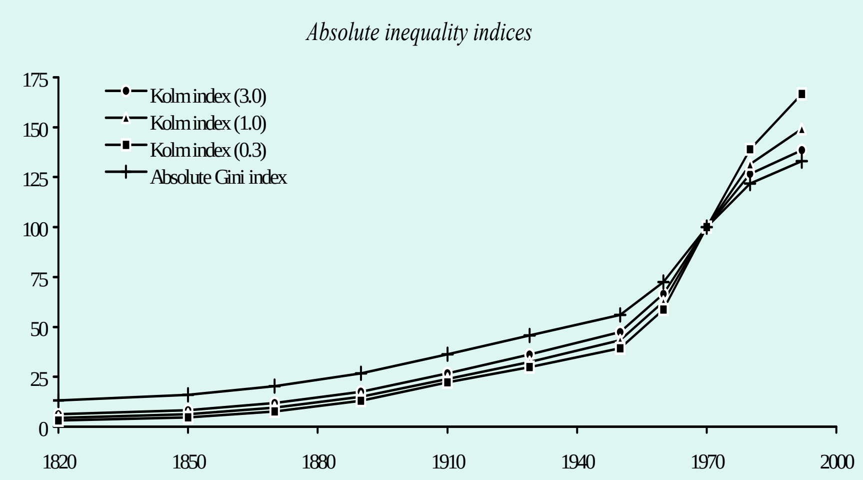
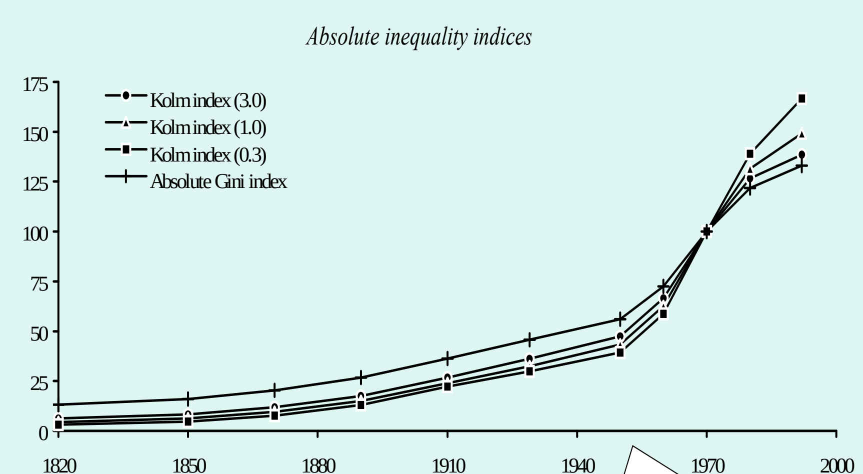
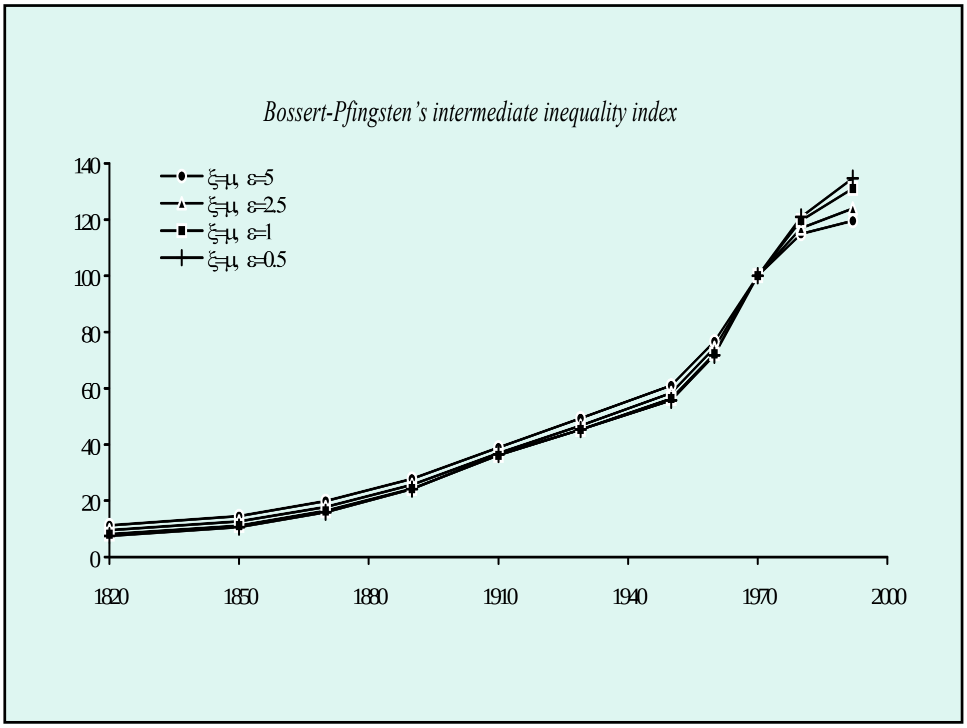
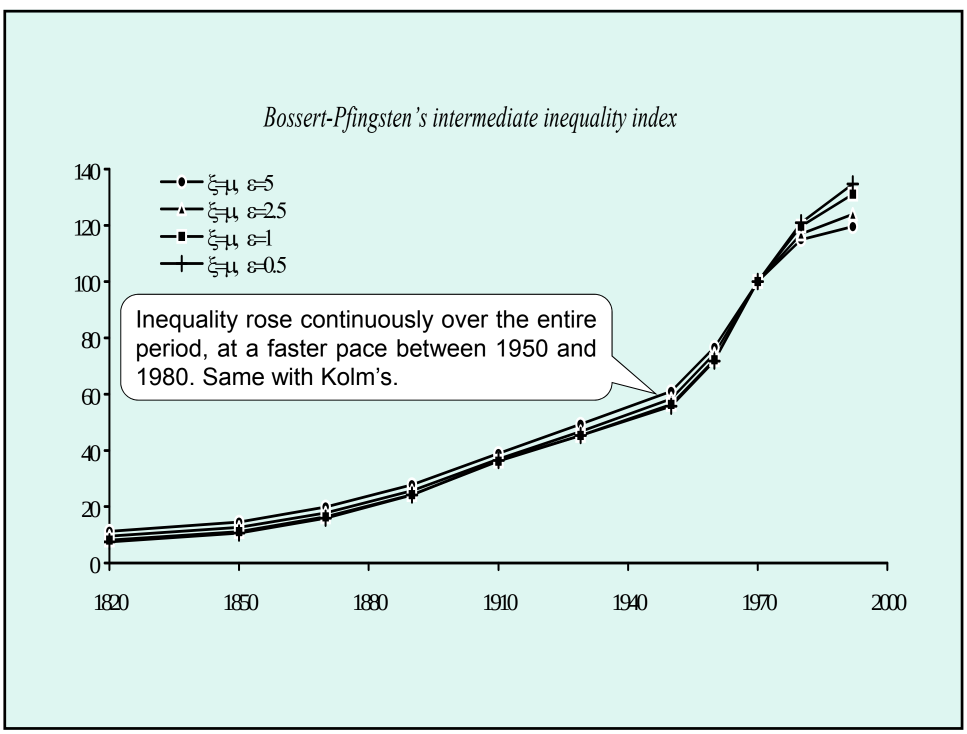
The rise of the Theil index is sharper during the 19th century, but it basically terminates by 1910.
Between 1970 and 1992, all three indices exhibit some modest widening of income disparities across world citizens. Accounting for the within-country distribution, however imperfectly, has therefore the effect of reversing the trend found earlier for international income inequality.





Inequality rose continuously over the entire period, at a faster pace between 1950 and 1980.


Bossert-Pfingsten's intermediate inequality index
ξ=μ, ε=5
ξ=μ, ε=2.5
ξ=μ, ε=1
ξ=μ, ε=0.5
140
120
100
80
60
40
20
0
1820
1850
1880
1910
1940
1970
2000


Bossert-Pfingsten's intermediate inequality index
ξ=μ, ε=5
ξ=μ, ε=2.5
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Inequality rose continuously over the entire period, at a faster pace between 1950 and 1980. Same with Kolm's.
140
120
100
80
60
40
20
0
1820
1850
1880
1910
1940
1970
2000

The secular movement of the world income distribution does not change whether we look at relative or non-relative measures – inequality has been rising.

The story is somewhat different, however, after the Second World War: the modest positive slope of relative inequality is matched by a steep ascent of absolute and intermediate inequality.

Conclusion: international inequality

The international distribution of real per capita GDP (i.e. ignoring within-country disparities) narrowed from 1970 to 2000 if we adopt a relative view of inequality;

it widened considerably if we assume an absolute or an intermediate conception, regardless of the index chosen and for most of the values of parameters.

Only the Bossert and Pfingsten's index for some combinations of the parameters suggests a fall of intermediate inequality.

Conclusion: global inequality

When we adjust for the within-country distribution of income, the evidence is almost unequivocally of a rise in income inequality from 1970 to 1992, whatever the underlying conception of inequality.

If we extend the time horizon to the whole post-war period, the results are more ambiguous, since the modest positive slope of relative inequality is matched by a steep ascent of absolute and intermediate inequality.

On a secular basis, from 1820 to 1992, the evidence is again one of a movement towards higher inequality both with relative and non-relative measures.

Inequality-Deprivation-Polarization-Social Exclusion

Income vs. Functionings

Symmetric sentiment vs. Asymmetric sentiment

In one period vs. Over time

Inequality

Income & Symmetric sentiment & in one period

An inequality index, which represents interpersonal income differences or spread of the distribution, is a function I defined from $\mathbb{D}$ to $\mathbb{R}^1$.

Inequality

The most well-known index of inequality is the Gini coefficient defined as:

$$G(x) = \frac{\frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j|}{2\lambda(x)}. \tag{1}$$

The numerator of (1) is the Gini mean difference. When divided by the mean $\lambda(x)$ it becomes the relative mean difference. Since

$$\min(x_i, x_j) = \frac{x_i + x_j - |x_i - x_j|}{2}, \tag{2}$$

we can rewrite $G^n(x)$ as

$$\begin{aligned} G^n(x) &= 1 - \tfrac{1}{n^2\lambda(x)} \sum_{j=1}^{n} \sum_{i=1}^{n} \min(x_i, x_j) \\ &= 1 - \tfrac{1}{n^2\lambda(x)} \sum_{i=1}^{n} (2(n-i)+1)\overline{x}_i. \end{aligned} \tag{3}$$

Inequality

Each individual feels alienated from others located at different points of the income scale:

$$A_i(x) = \sum_{j=1}^{n} |x_j - x_i|$$

if there is more than one individual with the same income level:

$$A_i(\pi, x) = \sum_{j=1}^{k} |x_j - x_i| \pi_j$$

Inequality

Income inequality, in the whole society, is the sum of these sentiments of alienation:

$$I(\pi, x) = \sum_{i=1}^{k} \pi_i A_i(\pi, x) = \sum_{i=1}^{k} \sum_{j=1}^{k} \pi_j \pi_i |x_j - x_i|$$

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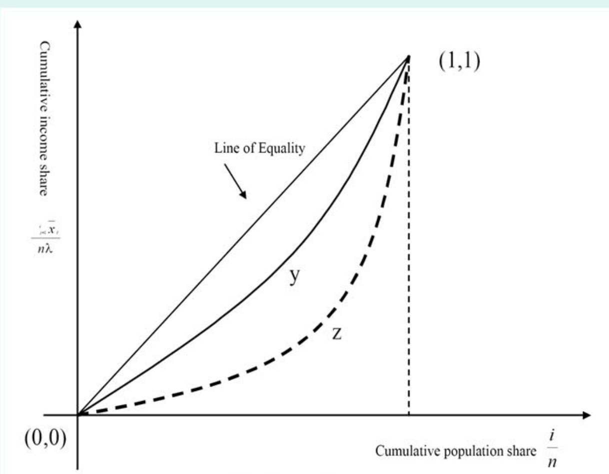
Proportional to Absolute Gini

Lorenz Curve



The Lorenz Curve

Polarization: the ER Approach

Income & Symmetric sentiment & in one period

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Polarization: the ER Approach

Each individual identifies with people having the same income, identification/alienation gives rise to effective alienation:

$$EA_i(\pi, x) = \pi_i^{\alpha} A_i(\pi, x)$$

Polarization, in the whole society, is the sum of these sentiments of effective alienation:

$$P(\pi, x) = \sum_{i=1}^{k} \pi_i EA_i(\pi, x) = \sum_{i=1}^{k} \sum_{j=1}^{k} \pi_i^{1+\alpha} \pi_j |x_j - x_i|$$

Polarization: the ER Approach

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$$EA$$

The Esteban-Ray (absolute) measure
The Duclos-Esteban-Ray measure

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Polarization

Polarization is different from inequality:

1. fails to satisfy Pigou-Dalton transfers principle.
2. global concept.

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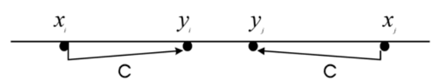

Figure 1: Progressive transfer.

Axiom 1 *: Pigou-Dalton Transfers Principle (PD). If $y \in \mathbb{D}^n$ is obtained from $x \in \mathbb{D}^n$ by a progressive transfer, then $I^n(y) < I^n(x)$.*

Thus, *PD* implies that a progressive transfer reduces inequality. Likewise, a regressive transfer increases inequality.

Polarization

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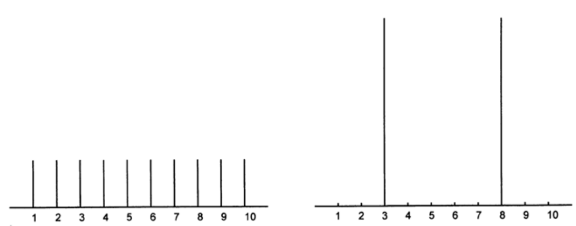


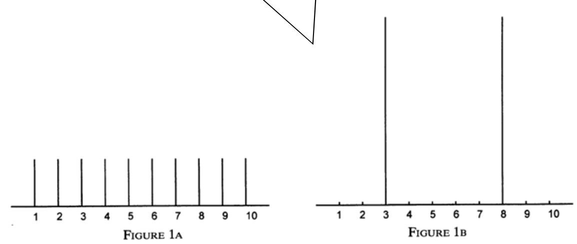
FIGURE 1A FIGURE 1B

Polarization

Polarization is different from inequality:

1. fails to satisfy Pig rs principle.
2. global conc

Inequality decreased
Polarization increased



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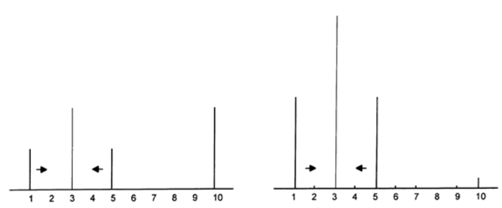

FIGURE 4.—Diagrams to illustrate Example 4.

Polarization

Polarization is different from inequality:

Inequality decreases in both.
Polarization increases in 4A and decreases in 4B.

1. fails to sati -Dalton transfers principle.
2. global concep

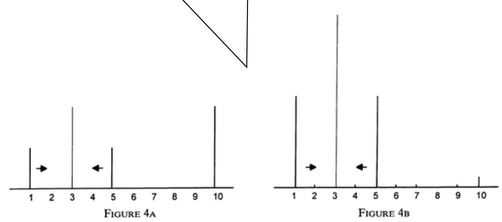

FIGURE 4.—Diagrams to illustrate Example 4.

Polarization

Alternative measures of polarization have been proposed in the literature following the **Wolfson**'s (1994) approach. Here polarization is “shrinkage of the middle class”, dispersion around the median of the distribution.

Polarization: the Wolfson's approach

Two characteristics that are regarded as being intrinsic to the notion of polarization:

1. increasing spread,
2. increasing bipolarity.

Polarization: the Wolfson's approach

According to increasing spread, a movement of incomes from the middle position to the tails of the income distribution increases polarization.

In other words, as the distribution becomes more spread out from the middle position, polarization increases.

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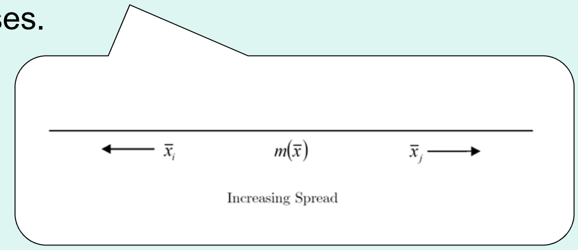

Polarization: the Wolfson's approach

On the other hand, increasing bipolarity means that a clustering of incomes below or above the median augment polarization.

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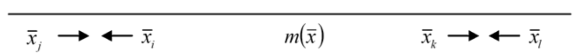

Polarization: the Wolfson’s approach

The measure of polarization in Wolfson (1994) can be rewritten as

$$P^W = \frac{\lambda}{m}\left[\frac{1}{2} - L - \frac{G}{2}\right],$$

where m stands for the median income, λ for the mean income, G for the Gini index and $L = L\left(\frac{1}{2}\right)$ for the value of the ordinate of the Lorenz curve at the median income.

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Class of indices by Wang and Tsui (JPET, 2000)

Polarization: the Wolfson's approach

Wang and Tsui (JPET, 2000) suggested the use of the following as absolute and relative indices of polarization respectively:

$$P_\Phi(x) = \frac{1}{n}\sum_{i=1}^{n}\Phi(d_i),$$

$$P_\Psi(x) = \frac{1}{n}\sum_{i=1}^{n}\Psi(D_i).$$

where:

$$d_i = |x_i - m(x)|,$$

and

$$D_i = \left|\frac{x_i - m(x)}{m(x)}\right|.$$

d_i is translation invariant while D_i is scale invariant. Φ and Ψ are increasing, strictly concave in $\mathbb{R}_+$ and $\Phi(0) = 0$ and $\Psi(0) = 0$.

Polarization curve

The (relative) polarization curve of any income distribution shows for any population proportion, how far the total income enjoyed by that proportion, expressed as a fraction of $nm(x)$, is from the corresponding income that would receive under the hypothetical distribution where everybody enjoys the median income.

For any $x \in \mathbb{D}$, the polarization curve (PC) ordinate corresponding to the population proportion $\frac{k}{n}$ $(1 \le k \le \overline{n})$ is $P\left(x, \frac{k}{n}\right) = \frac{1}{nm(x)} \sum_{k \le i \le \overline{n}}^{n} (m(x) - x_i)$, and corresponding to the population proportion $\frac{k}{n}$, $(\overline{n} \le k \le n)$ this ordinate is $P\left(x, \frac{k}{n}\right) = \frac{1}{nm(x)} \sum_{\overline{n} \le i \le k}^{n} (x_i - m(x))$, where $\overline{n} = \frac{n+1}{2}$.

Note that the ordinate at $\frac{\overline{n}}{n}$ involves the income level $x_{\overline{n}} = m(x)$. Now, if n is odd, $m(x)$ is one of the incomes in the distribution. However, for even n, $x_{\overline{n}}$ is not in x, we define the ordinate at $\frac{\overline{n}}{n}$, since in polarization measurement, the median income is the reference income.

Polarization curve

Example 3 : *For the distributions* $x = (1, 3, 5, 9, 11)$, $m(x) = 5$, $x_- = (1, 3)$, $x_+ = (9, 11)$. *The ordinates of the polarization curve are:*

$P\left(x, \frac{1}{5}\right) = \frac{1}{25}((5-1) + (5-3)) = \frac{6}{25}$;

$P\left(x, \frac{2}{5}\right) = \frac{1}{25}((5-3)) = \frac{2}{25}$;

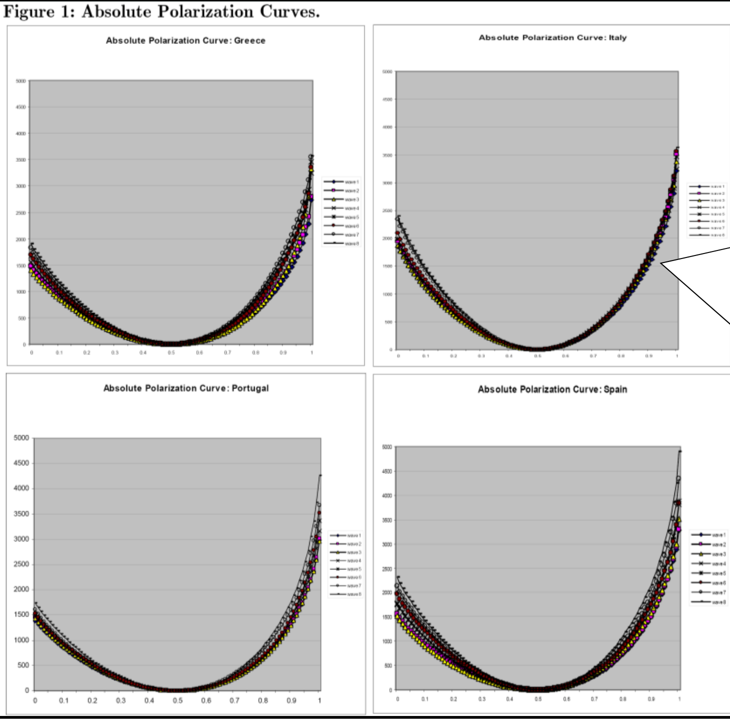
$P\left(x, \frac{3}{5}\right) = 0$;

$P\left(x, \frac{4}{5}\right) = \frac{1}{25}((9-5)) = \frac{4}{25}$;

$P\left(x, \frac{5}{5}\right) = \frac{1}{25}((9-5) + (11-5)) = \frac{10}{25}$.

For a typical income distribution x, up to $\frac{\overline{n}}{n}$, the polarization curve decreases monotonically, at $\frac{\overline{n}}{n}$ it coincides with the horizontal axis and then it increases monotonically. If x is an equal distribution, then the curve becomes the horizontal axis itself.

Figure 1: Absolute Polarization Curves.



An asymmetry in distances from the median exists in all cases.

This observation is a consequence of the longer right tail of the curves.

Polarization: the Wolfson's approach

Definition 2 : *Given any two income distributions $x, y \in \mathbb{D}$, x is said to dominate y with respect to polarization, which we write xPy if the polarization curve of x is nowhere below that of y, and at some places above.*

Theorem 11 : *Let $x,y \in \mathbb{D}$ be arbitrary. Then the following statements are equivalent:*
1) xPy;
2) $P(x) > P(y)$ for all relative polarization indices belonging to the class of Wang and Tsui.

This theorem indicates that an unambiguous ranking of income distribution can be obtained if and only if their polarization curves do not intersect.

Deprivation

The definition of relative deprivation adopted is the following:

"We can roughly say that [a person] is relatively deprived of X when

(i) he does not have X,

(ii) he sees some other person or persons, which may include himself at some previous or expected time, as having X,

(iii) he wants X, and

(iv) he sees it as feasible that he should have X"

(Runciman, 1966, p.10).

Runciman further adds: "The magnitude of relative deprivation is the extent of the difference between the desired situation and that of the person desiring it".

Runciman further adds: “The magnitude of relative deprivation is the extent of the difference between the desired situation and that of the person desiring it”.

One of the key variables in measuring deprivation is the **reference group**, that is the group with which a person compares itself.

The measurement of deprivation in a society has traditionally been conducted analyzing **incomes** of individuals, as income summarizes command over resources and is an index of the individual’s ability to consume commodities.

In this framework a seminal paper is that by **Yitzhaki (1979)** where it is suggested that an appropriate index of aggregate deprivation is the **absolute Gini index**.

A reason for being interested in deprivation is its representation of the degree of **discontent** or **injustice** felt by the members of a society.

In view of this fact, **Podder (1996)** criticizes the measure of deprivation proposed in the literature: **deprivation and inequality are different concepts**, hence an index of inequality, such as the **Gini coefficient, is inappropriate to measure deprivation.**

In **Podder** (1996) the distinction between the two is explained by their relations to **envy**.

In **Podder** (1996) the distinction between the two is explained by their relations to **envy**.

"We say that a person *i* has a feeling of envy towards person *j* if he prefers to **exchange his consumption bundle** with that of person j".

Deprivation is proportional to the feeling of envy towards the better off.

Equity—the absence of inequality—is the absence of envy in all economic agents. At the same time, **equity coincides with minimum deprivation**—all individuals possess the same level of income.

In constrast, the **upper bounds of deprivation and inequality do not coincide**.

Equity—the absence of inequality—is the absence of envy in all economic agents. At the same time, **equity coincides with minimum deprivation**—all individuals possess the same level of income.

In constrast, the **upper bounds of deprivation and inequality do not coincide**.

Maximum **inequality is reached when one individual monopolizes** the entire total income; **maximum deprivation**, on the other hand, is obtained when the **society is polarized in two equal-sized groups**, those possessing income and those not possessing it.

An analogous distinction with inequality is at the basis of the concept of **polarization of Esteban and Ray (1994).**

Following the Esteban and Ray **identification/alienation framework, Bossert, D'Ambrosio and Peragine (2007)** proposed an alternative index of deprivation.

Deprivation

Income & Asymmetric sentiment & in one period

Each individual feels deprived only in comparison with others located at higher points of the income scale:

$$
\begin{aligned}
d_i(x) &= (x_j - x_i) \quad \text{if } x_i < x_j \\
&= 0 \quad \text{else}
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Comparison with others located at lower points of the income scale gives rise to "Satisfaction"

Deprivation

Total deprivation felt by an individual is:

$$D_i(x) = \sum_{j=i+1}^{k} (\overline{x}_j - \overline{x}_i)\,\pi_j$$

$$D_i(x) = \frac{\sum_{j=i+1}^{n} (\overline{x}_j - \overline{x}_i)}{n}$$

Deprivation, in the whole society, is the sum of these sentiments:

$$D(x) = \sum_{i=1}^{k} \sum_{j=i+1}^{k} (\overline{x}_j - \overline{x}_i)\,\pi_j \pi_i$$

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The Yitzhaki measure
which is equal to the Absolute Gini

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Deprivation

Other measures have been proposed in the literature based on income share differentials (Chakravarty, 1997, Kakwani, 1984), known as mesures of **relative deprivation**. Kakwani introduces the relative deprivation curve. The area under the deprivation curve is the **Gini coefficient**, the index of relative deprivation.

$D_i($

$$D_i(x) = \frac{\sum_{j=i+1}^{n}(\overline{x}_j - \overline{x}_i)}{n}$$

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Deprivation ... he sum of these sentiments:

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Deprivation curve

Kakwani (1984) introduced the relative deprivation curve. The area under the deprivation curve is the **Gini coefficient**, the index of relative deprivation.

Deprivation curve

Following Chakravarty, the total relative deprivation felt by an individual is:

$$D_i^r(x) = \frac{\sum_{j=i+1}^{n}(\overline{x}_j - \overline{x}_i)}{n\lambda(x)} .$$

We can rewrite $D_i^r(x)$

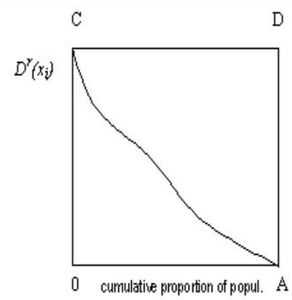
Ordinate of Lorenz Curve

$$D_i^r(x) = 1 - L(x_i) - \frac{(n-i)\,\overline{x}_i}{n\lambda(x)},$$

Deprivation curve

Kakwani defines the relative deprivation curve corresponding to the distribution x as the plot of $D_i^r(x)$ against the cumulative proportion of population $\frac{i}{n}(0 \leq i \leq n)$ and $D^r(x_0) = 1$. The relative deprivation curve is downward sloping but no definite conclusion can be drawn regarding its curvature (See Chakravarty et al., 1995).

Deprivation



The relative deprivation curve.

If the Lorenz curve coincides with the egalitarian line, then the relative deprivation curve coincides with the horizontal line OA. On the other hand, if there is maximum inequality, the curve coincides with CD.

Bossert, D'Ambrosio, Peragine

Deprivation: BDP

Functionings & Asymmetric sentiment & in one period

A deprivation score, q_i, is constructed for each population member, i, indicating the degree to which functionings that are considered relevant are not available to the agent.

Deprivation: BDP

Functionings & Asymmetric sentiment & in one period

A deprivation score, q_i, is constructed for each population member, i, indicating the degree to which functionings th… are considered relevant are not available to t… ent.

q_i is the functioning failure of individual i.
q_i's constitute the primary inputs of the analysis.

Deprivation: BDP

Each individual feels alienated only in comparison with others with less functioning failures.

Bossert, D'Ambrosio & Peragine (BDP)

The members of the class of deprivation measures, $D_i\colon \Omega \to \mathbb{R}_+$, characterized by BDP are such that the degree of deprivation for a distribution (π, q) is obtained as the product of two terms with the following interpretation. The first factor is a multiple of the ratio of the number of agents who have fewer functioning failures than i and the population size. This number is interpreted as an inverse indicator of agent i's capacity to identify with other members of society–the lack of identification. The second factor is the average of the differences between q_i and the functioning failures of all agents having fewer functionings failure than i. This part captures the aggregate alienation experienced by i with respect to those who are better off. In particular the index is defined by:

$$D_i(\pi, q) = \left(\sum_{j=1}^{i-1} \pi_j\right) \sum_{j=1}^{i-1} (\overline{q}_i - \overline{q}_j)\pi_j,$$

for all $(\pi, q) \in \Omega$.

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Deprivation: BDP

Deprivation, in the whole society, is the sum of these sentiments:

The BDP aggregate measure of deprivation is a function $\mathbf{D}\colon \Omega \to \mathbb{R}_+$ such that:

$$\mathbf{D}(\pi, q) = \sum_{i=1}^{K} \pi_i \left(\sum_{k=1}^{i-1} \pi_k \right) \sum_{j=1}^{i-1} (\overline{q}_i - \overline{q}_j)\pi_j,$$

for all $(\pi, q) \in \Omega$.

What about time?

Does individual well-being depend on the individual's history?

Does it depend on other individuals' histories?

Deprivation: Bossert and D'Ambrosio (BD)

BD introduce a one-parameter class of dynamic individual deprivation measures.

BD modify Yitzhaki's index to take into account the part of deprivation generated by an agent's observation that others in it reference group move on to a higher level of income than himself.

The parameter reflects the relative weight given to these dynamic considerations, and the standard Yitzhaki index is obtained as a special case.

BD formalize an additional idea of Runciman that has not been explored in the literature yet:

"The more the people a man sees promoted when he is not promoted himself, the more people he may compare himself with in a situation where the comparison will make him feel relatively deprived" (Runciman, 1966, p.19).

Relative deprivation of an individual in BD framework is determined by the interaction of two components:

1. the average gap between the individual's income and the incomes of all individuals richer than him (the traditional way of measuring individual deprivation);
2. a function of the number of people who were ranked below or equal in the previous-period distribution but are above the person under consideration in the current distribution.

BD use an axiomatic approach to derive classes of indices that capture these ideas.

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BD use ... ses of indices t...

For all $(x^0, x^1) \in \mathbb{R}_+^{2n}$, where $\alpha \in [1, \infty)$ is a parameter,

$$D_i^\alpha(x^0, x^1) = \frac{\alpha^{|\text{number of people that passed}|}}{n} \sum_{j=i+1}^{n} (\overline{x}_j^1 - \overline{x}_i^1).$$

- Deprivation has attracted increasing attention in the past decades when the measurement of individual well-being gained importance not only in the academic context but also in the public discourse and in policy-making circles.

- The main reason for this is the characteristic at the basis of the concept: the observation that, since individuals do not live in isolation, they determine their well-being also from comparisons with others. Comparisons to richer individuals matter.

- Although this consideration appears to be absent from much of standard economic modeling, it has been shown to be one of the main determinants of self-reported satisfaction with income and life. For a survey see, for example, Frey and Stutzer (2002).

Measuring relative deprivation is important not only per se but also because of its links to major social phenomena such as:

- crime (Stack, 1984),

- political violence (Gurr, 1968),

- health status (Wagstaff and van Doorslaer, 2000; Jones and Wildman, 2008),

- mortality (Salti, 2010);

- migration decisions (Stark and Taylor, 1989).

Application

The paper with Frick explores the determinants of individual well-being as measured by self-reported levels of satisfaction with income and life.

Making full use of the panel data nature of the German Socio-Economic Panel, we provide empirical evidence for well-being depending on **absolute** and on **relative** levels of income in a **dynamic framework**.

DF propose a **new functional** form to represent **interdependence of preferences** over income distributions, that is, an individual's preferences that depend jointly on the entire distribution of income, and use data from **Germany** over the period 1992 to 2007 to **test its validity**.

A Dynamic-Status-Concerned Utility Function

The focus of the income distribution literature has been on measuring (income) deprivation and satisfaction.

(Interdependent) preferences only appear implicitly in the previous literature, where **it is assumed that well-being of an individual depends negatively on relative deprivation and positively on relative satisfaction.**

Experimentalists, on the other hand, have proposed alternative specifications of utility functions and make use of interdependence in preferences to explain the behavior of subjects that repeatedly violate the game theoretical predictions.

Deprivation and satisfaction are very similar to the concepts of disadvantageous and advantageous inequality of Fehr and Schmidt's (1999) individual utility function, defined by:

$$U_i(y) = y_i + \alpha \frac{\sum_{j \in B_i(y)} (y_j - y_i)}{n} + \beta \frac{\sum_{j \in W_i(y)} (y_i - y_j)}{n},$$

where $\alpha \leq \beta \leq 0$ are parameters

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Disadvantageous inequality / relative deprivation

Advantageous inequality / relative satisfaction

Deprivation and satisfaction are very similar to the concepts of disadvantageous and advantageous inequality of Fehr and Schmidt's (1999) individual utility function, defined by:

$$U_i(y) = y_i + \alpha \frac{\sum_{j \in B_i(y)} (y_j - y_i)}{n} + \beta \frac{\sum_{j \in W_i(y)} (y_i - y_j)}{n},$$

where $\alpha \leq \beta \leq 0$ are parameters

Disadvantageous inequality / relative deprivation

Advantageous inequality / relative satisfaction

According to Fehr and Schmidt, individuals dislike inequitable distributions. "They experience inequity if they are worse off in material terms than the other players in the experiment, and they also feel inequity if they are better off. (...) (H)owever, we assume that, in general, subjects suffer more from inequity that is to their material disadvantage than from inequity that is to their material advantage."(Fehr and Schmidt, 1999, p.822.).

D’Ambrosio and Frick (2012) test FS and add concerns for history when making assumptions about individual utility.

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The functional form:

Well-being of an individual as measured by the **degree of personal satisfaction with respect to own income** depends at time *t* on four components.

i) The **absolute component**, that is, the standard of living of the individual at time *t*;

ii) the **relative component**, that is, the income of the individual compared to that of others at the same time *t*.

Both components have a **dynamic counterpart**:

iii) the **absolute dynamic component**, that is, how the individual performed in terms of **own income** from time $t - 1$ to time t;

iv) the **relative dynamic component**, that is, how the individual performed from $t - 1$ to t with respect to **others’ incomes**.

The dynamic components

The dynamic components aim at capturing the effects of **history, both of the individual and of others**.

One's **own history** is clearly relevant to one's well-being, because personal history is a major determinant of aspiration levels and own standards of living.

We hypothesize that the **history of others** will also have an impact on one's well-being, above and beyond one's relative standing in society.

Own history

The **absolute dynamic component** focuses on own history.

Only an increase in income is expected to have a positive effect on income satisfaction.

The dynamic components: The history of others

Specifically, well-being depends not only on one's ranking in society in the past and at present.

It can also depend on the situation of other individuals populating the income curve: if another individual, who **used to be behind** in terms of income, succeeded in **moving ahead**, one's well-being might be affected differently as compared to a situation in which the income ordering has been preserved.

The history of others

An individual concerned with status might be **satisfied if he was able to pass others** and might show **disappointment with his income if others were able to pass him**, in a way that will not be captured by his relative status in past and present income distributions.

The history of others

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This sentiment, captured by the **relative dynamic component**, is in addition to that embedded in the absolute and relative components of well-being: somebody who earns a lot at time *t* and is higher up in the income scale at time *t* might still show disappointment if others were able to pass him and he was not able to pass anyone.

The Utility Function

$$U_i^t(y^{t-1}, y^t) = \underbrace{\tau y_i^t}_{\text{i) Abs.}} + \underbrace{\vartheta \frac{y_i^t - y_i^{t-1}}{y_i^{t-1}}}_{\text{ii) Abs.Dyn.}} \underbrace{+\kappa \frac{\sum\limits_{j \in BB_i(y^t)} (y_j^t - y_i^t)}{n\lambda(y^t)} + \chi \frac{\sum\limits_{j \in WW_i(y^t)} (y_i^t - y_j^t)}{n\lambda(y^t)}}_{\text{iii) Rel.}}$$
$$\underbrace{+\varepsilon \frac{\sum\limits_{j \in WB_i(y^t)} (y_j^t - y_i^t)}{n\lambda(y^t)} + \eta \frac{\sum\limits_{j \in BW_i(y^t)} (y_i^t - y_j^t)}{n\lambda(y^t)}}_{\text{iv) Rel.Dyn.}} \tag{7}$$

where $\tau, \vartheta, \kappa, \chi, \varepsilon, \eta$ are parameters indicating the weight on the individual's utility of alternative income specifications.

The literature

The role of an **individual's history** in measuring well-being is contained also in **Gilboa and Schmeidler (2001)** but with a different perspective from DF.

Their setting is more similar to **habit formation** (Pollak, 1970) than to the dynamic components DF introduced.

"The individual's own history of payoffs affects her aspirations. For instance, when an individual is accustomed to a certain standard of living, her well-being depends mostly on deviations from it."

Well-being depends on the instantaneous payoff defined as the difference between the objective payoff and the individual's aspiration.

The literature

The role of **histories of others** in measuring well-being appeared in **Hirschman (1973)**, labeled as the **tunnel effect**.

"Suppose that the individual has very little information about his future income, but at some point a few of his relatives, neighbors, or acquaintances improve their economic or social position. Now he has something to go on: expecting that his turn will come in due course, he will draw gratification from advances of others-for a while."

In Hirschman, though, the temporal aspect of the concept of history is somehow lost when **advances of others are simply considered as the presence of richer** individuals, giving rise to **inequality in the present distribution of income.**

The literature

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Tunnel Effect /Signal Effect (+ coeff) vs Status Effect (- coeff)

In DF opinion, advances of similar individuals are better captured by the relative dynamic component DF propose.

The links with subjective well-being

Generally, subjective well-being is measured by interviewing people in surveys using a single-occasion, self-report question.

Papers on this subject make use of both cross-sectional data (e.g. Eurobarometer Surveys, United States General Social Survey), and panel data (e.g. the German Socio-Economic Panel (SOEP), the British Household Panel Survey and the European Community Household Panel).

D'Ambrosio and Frick (2012) investigate the relationship between subjective well-being using panel data since the latter **allow to control for otherwise unobserved individual characteristics**. This is especially important if these unobservables are systematically correlated with reported subjective well-being.

The measure of subjective well-being in the SOEP, i.e. `satisfaction with income‘ or `satisfaction with life‘, is measured on an 11-point scale, ranging from 0 (`completely dissatisfied') to 10 (`completely satisfied').

The data used covers the period 1990 (the first data available for the East German sample) to 2007 (the most recent available data when the paper was written).

The overall sample contains all adult respondents with valid information on income satisfaction, that is approximately 184,000 observations based on 27,200 individuals in East and West Germany.

The Estimation Method

We estimate fixed-effects regression model, assuming linearity.

We also run a random-effects model in order to investigate the effects of time invariant control variables, such as gender and migration status.

The Results: Ysat

ABS.	0.091**	0.055**	0.054**
	(0.004)	(0.006)	(0.006)
Deprivation	-0.877**	-	-
	(0.031)	-	-
Satisfaction	0.353**	-	-
	(0.018)	-	-
ABS. DYN.	-	0.437**	-
	-	(0.014)	-
ABS. DYN.: Positive % change	-	-	0.168**
	-	-	(0.020)
ABS. DYN.: Negative % change	-	-	-1.039**
	-	-	(0.035)
REL.: Deprivation	-	-1.151**	-1.126**
	-	(0.039)	(0.039)
REL. DYN.: Deprivation	-	0.659**	0.711**
	-	(0.114)	(0.114)
REL.: Satisfaction	-	0.462**	0.454**
	-	(0.022)	(0.022)
REL. DYN.: Satisfaction	-	-1.305**	-1.195**
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The absolute dynamic component has the expected signs, positive for those experiencing an income growth, negative otherwise. Losses have a greater effect than gains, confirming the presence of loss aversion.

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Germans are satisfied with respect to poorer individuals and feel deprived when compared to richer ones only when the comparison takes place with respect to individuals that are and were ahead or behind in both years (REL. deprivation and REL. satisfaction). Germans are interested in keeping their status: being still richer than the same individuals increases satisfaction and being still poorer has the reverse effect.

ABS.	0.091**	0.055**	0.054**
	(0.004)	(0.006)	(0.006)
Deprivation	-0.877**	-	-
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Satisfaction	0.353**	-	-
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The sign of the coefficients reverse for satisfaction with respect to passers and passees, indicating that signal has an additional role together with status. The comparision with those that are behind but were ahead in the previous period (REL. DYN satisfaction) has a negative effect on Germans' satisfaction with income or life. This fact can be interpreted as containing a negative information, signalling to the individual that he could be one of them tomorrow.

ABS.	0.091**	0.055**	0.054**
	(0.004)	(0.006)	(0.006)
Deprivation	-0.877**	-	-
	(0.031)	-	-
Satisfaction	0.353**	-	-
	(0.018)	-	-
ABS. DYN.	-	0.437**	-
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For satisfaction with income, the coefficient of the relative dynamic deprivation component (REL. DYN. deprivation) is positive. Germans do not prove any feeling of deprivation with respect to individuals who have passed them, actually, being passed makes them more satisfied with their income. Being passed is seen as good auspice for future gains. For life satisfaction, the coefficient of the relative dynamic deprivation component (REL. DYN. deprivation) is not significant.

Conclusion

People's satisfaction with income and life depends on what they observe around them and on the histories of themselves and the others.

The separation of the relative income performance with respect to richer individuals in two components has the advantage of reconciling two views – status vs. signal - that were, so far, considered in opposition in the literature.

Conclusion

Both status and signal influence individual well-being.

Germans enjoy keeping their status, that is, being constantly richer increases income satisfaction and being constantly poorer has the opposite effect;

At the same time, the presence of newly richer and poorer individuals plays the informational role described in Hirschman’s tunnel effect.

While controlling for the absolute and relative components, passing signals to the individual that he could be passed tomorrow (it decreases satisfacfaction) and being passed signals that he could pass tomorrow (it increases satisfaction).

The intensity of deprivation

Some authors who deal with individual deprivation focus on the task of capturing the intensity of deprivation felt by an individual in the comparison to those who are better off by enriching measures that are based on income shortfalls.

Among other features, their contributions can be viewed as addressing the feasibility aspect of deprivation underlined by Runciman (1966).

According to Runciman (1966, p.10):

“[t]he qualification of feasibility is obviously imprecise, but it is necessary in order to exclude fantasy wishes. A man may say with perfect truth that he wants to be as rich as the Aga Khan [...] but to include these under the heading of relative deprivation would rob the term of its value.”

The intensity of deprivation

A similar position on feasibility can be found in Gurr (1968, p.1104) who states that:

“[r]elative deprivation is defined as actors’ perceptions of discrepancy between their value expectations (the goods and conditions of the life to which they believe they are justifiably entitled) and their value capabilities (the amounts of those goods and conditions that they think they are able to get and keep).”

Operationalize feasibility: limit comparison group

The question of how to deal with the feasibility aspect is a subtle issue.

One possible response is to simply reduce Yitzhaki’s (1979) proposed comparison group of all richer individuals by eliminating individuals who are ‘much richer’ (such as the Aga Khan in the above Runciman quote) altogether.

However, such a rather drastic move would seem to have problems of its own.

First: how much richer is hard to define properly.

Second, we do not want to exclude the much richer entirely from consideration.

Operationalize feasibility: more significance to closer individuals

Operationalize feasibility:

A more adequate response that myself and Bossert (2014) (along with all other relevant contributions that we are aware of) endorse is to find a way of assigning more significance to a richer individual depending on how close her income is to that of the person under consideration.

We depart significantly from the earlier literature by retaining a structure that is based on income shortfalls.

Operationalize feasibility: other relevant contributions

Contributions that are close to our own as far as the feasibility issue is concerned include:

- Paul (1991),
- Chakravarty and Chattopadhyay (1994),
- Podder (1996)
- Esposito (2010), which is the only one that provides a characterization of the individual deprivation index that is being proposed.

Operationalize feasibility: other relevant contributions

All of these authors abandon the income shortfall approach in the sense that they either operate within a utility shortfall framework as that mentioned in Hey and Lambert (1980) or focus on income ratios rather than income differences.

With Bossert we show that these modifications are not necessary in order to address the feasibility problem: to ensure that higher incomes have a higher impact on individual deprivation the closer they are to the income of the individual in question, the income shortfall approach can be retained.

We provide a characterization of a class of individual indices with this property in addition to axiomatizing a more general class.

The Hey and Lambert linear income shortfall index

The **linear income shortfall deprivation measure** D^L proposed by Hey and Lambert (1980) and inspired by Yitzhaki (1979) is defined as follows. For all $(\mathbf{y}; x) \in \mathbb{R}_+^{n+1}$,

$$D^L(\mathbf{y}; x) = \begin{cases} 0 & \text{if } B(\mathbf{y}; x) = \emptyset, \\ \sum_{j \in B(\mathbf{y}; x)} \frac{1}{n}(y_j - x) & \text{if } B(\mathbf{y}; x) \neq \emptyset. \end{cases}$$

The generalized income shortfall index

This index can be generalized in an intuitive manner.

For any **increasing function** $F\colon \mathbb{R}_{++} \to \mathbb{R}_{++}$, the corresponding individual deprivation index D^F is defined by letting, for all $(\mathbf{y}; x) \in \mathbb{R}_+^{n+1}$,

$$D^F(\mathbf{y}; x) = \begin{cases} 0 & \text{if } B(\mathbf{y}; x) = \emptyset, \\ \sum_{j \in B(\mathbf{y};x)} F(y_j - x) & \text{if } B(\mathbf{y}; x) \neq \emptyset. \end{cases}$$

These indices generalize D^L, which is the special case obtained by choosing $F(t) = t/n$ for all $t \in \mathbb{R}_{++}$ in the definition of D^F.

Consequently, we refer to the members of this class as **generalized income shortfall deprivation measures**.

Paul's (1991) index

Paul (1991) proposes an individual index that is sensitive to transfers among richer individuals.

It is obtained by considering specific increasing transformations of the **ratios** x/y_j as opposed to the income shortfalls $y_j - x$.

His class uses a parameter $\beta \in (1, \infty)$ that reflects the degree of sensitivity to income transfers among the better-off. For any $\beta \in (1, \infty)$, the index D^β is defined by letting, for all $(\mathbf{y}; x) \in \mathbb{R}_+^{n+1}$,

$$D^\beta(\mathbf{y}; x) = \begin{cases} 0 & \text{if } B(\mathbf{y}; x) = \emptyset, \\ \sum_{j \in B(\mathbf{y};x)} \frac{1}{n} \left(\frac{x}{y_j}\right)^\beta - \frac{1}{n}|B(\mathbf{y}; x)| & \text{if } B(\mathbf{y}; x) \neq \emptyset. \end{cases}$$

Chakravarty and Chattopadhyay's (1994) and Podder's (1996) indices

The indices of Chakravarty and Chattopadhyay (1994) and of Podder (1996) are special cases of the class D^U, where $U\colon \mathbb{R}_+ \to \mathbb{R}_+$ is an increasing and strictly concave utility function.

This class of measures was suggested by Hey and Lambert (1980) and the individual index corresponding to any such U is defined by letting, for all $(\mathbf{y}; x) \in \mathbb{R}_+^{n+1}$,

$$D^U(\mathbf{y}; x) = \begin{cases} 0 & \text{if } B(\mathbf{y}; x) = \emptyset, \\ \sum_{j \in B(\mathbf{y}; x)} [U(y_j) - U(x)] & \text{if } B(\mathbf{y}; x) \neq \emptyset. \end{cases}$$

Chakravarty and Chattopadhyay (1994) employ a power function U with a positive power, whereas Podder (1996) suggests to use a logarithmic utility function U.

Esposito's (2010) index

Esposito (2010), following these earlier contributions, proposes and characterizes a class based on relative utility gaps, employing a power function with a positive power α, defined by $U(t) = t^\alpha$ for all $t \in \mathbb{R}_+$.

The class of individual deprivation measures of Esposito (2010) is D^α with a parameter $\alpha \in \mathbb{R}_{++}$ defined by letting, for all $(\mathbf{y}; x) \in \mathbb{R}_+^{n+1}$,

$$D^\alpha(\mathbf{y}; x) = \begin{cases} 0 & \text{if } B(\mathbf{y}; x) = \emptyset, \\ \sum_{j \in B(\mathbf{y}; x)} \frac{1}{n} \frac{(y_j^\alpha - x^\alpha)}{y_j^\alpha} & \text{if } B(\mathbf{y}; x) \neq \emptyset. \end{cases}$$

What is common to the classes D^{β}, D^{U} and D^{α} is that they cannot be expressed as functions of the income shortfalls $y_j - x$.

Thus, these measures **accommodate the feasibility aspect of individual deprivation by deviating from the linearity** exhibited in D^{L} *and* **from the notion that individual deprivation is based on income shortfalls**. In contrast, the measures we advocate—the subclass of D^{F} corresponding to **increasing and strictly concave functions** F—retain the traditional reliance on income shortfalls, thereby illustrating that the desire to incorporate **feasibility issues does not require the income shortfall approach to be abandoned** altogether.

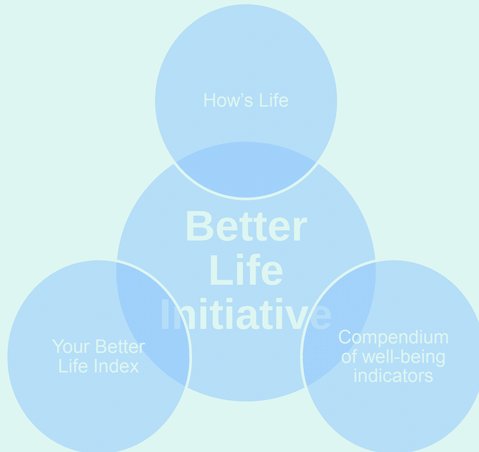
Our index: the concave function

Clearly, any strictly concave function $F\colon \mathbb{R}_{++} \to \mathbb{R}_{++}$ can be used to generate an individual deprivation index that belongs to the subclass characterized in Theorem 2.

Prominent examples include functions of the form $F(t) = t^{\alpha}/n$ for all $t \in \mathbb{R}_{++}$ where the power α is in the interval $(0, 1)$ to ensure that the resulting function is strictly concave.

For instance, the square root multiplied by $1/n$ is obtained for $\alpha = 1/2$.

OECD Better Life Initiative



Better policies for better lives

OECD Better Life Initiative

Your Better Life Index

An interactive tool to compare countries according to the things that matter to YOUR life

It covers 34 countries and 11 dimensions of well-being

It is a gateway to OECD work

OECD Better Life Initiative

Your Better Life Index

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